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Hydraulics of Great Lakes Inlets

by
William N. Seelig and Robert M. Sorensen

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Reversing currents in inlets on the Great Lakes are generated primarily by long wave seiching modes of the lakes rather than by the astronomical tide. Field measurements were conducted in 1974-75 at nine harbors on the Great Lakes to: (a) Investigate the nature of long wave excitation and the generating mechanism for significant inlet velocities, (b) establish techniques for predicting inlet-bay system response, and (c) develop base data for future planning and design studies. Data collected include continuous harbor water (continued)		

level measurements at all sites, inlet velocity measurements at the primary site (Pentwater, Michigan), and channel hydrographic surveys at the sites where more recent data were needed. Available historic water level and velocity data for some of the harbor sites were also used.

Amplified harbor oscillations and generation of the highest inlet velocities are caused by the Helmholtz resonance mode which has a period of 0.6 to 5 hours for the inlet-bay systems studied. A recently developed, simple numerical model is shown to be effective in predicting inlet-bay response over the range of excitation periods encountered. A finite-difference form of the continuity equation is shown to adequately predict inlet velocities if high-quality bay water level records are available. Selected data from the study sites are presented to demonstrate the hydraulic response of the inlet-bay systems and the applicability of the prediction schemes. Examples to demonstrate use of the concepts and techniques developed in the study are applied to the design of a new inlet channel and to the modification of an existing channel.

PREFACE

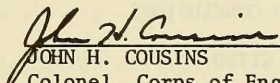
This report is published to provide coastal engineers with an analysis of the hydraulic response of inlet-bay systems on the Great Lakes. The work was carried out under the coastal research program of the U.S. Army Coastal Engineering Research Center (CERC).

The report was prepared by William N. Seelig and Dr. Robert M. Sorensen, Coastal Structures Branch, Research Division, CERC, under the general supervision of R.P. Savage, Chief, Research Division.

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Comments on this publication are invited.

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JOHN H. COUSINS
Colonel, Corps of Engineers
Commander and Director

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CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.8532	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.1745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins ¹

¹To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9) (F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9) (F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

A_b	inlet cross-sectional area at the bay end
A_{bay}	bay surface area
A_c	inlet cross-sectional area
A_{ij}	grid cross-sectional area
A_s	inlet cross-sectional area at the sea end
B	inlet width
B_{ij}	grid cell width
c	wave speed
D_{ij}	grid cell depth
d	water depth
g	acceleration due to gravity
h	water surface elevation
h_b	water surface elevation in the bay
h_b'	water surface elevation in the bay at the previous time step
h_s	water surface elevation in the sea
IC	number of channels in the grid
IS	number of sections in the grid
i	grid cell subscript indicating the channel
j	grid cell subscript indicating the cross section
k	mode of oscillation
L	inlet length
L'	added inlet length in the acceleration terms to account for long wave radiation from the harbor
L_b	basin length
L_{ij}	grid cell length

SYMBOLS AND DEFINITIONS--Continued

M	number of inlets connecting a bay to a sea
n	Manning's n
$n_{i,j}$	Manning's n of a grid cell
Q	discharge
Q_m	discharge for the mth inlet
Q_T	total discharge for all inlets
r	hydraulic radius of a channel
T_H	inlet-bay system Helmholtz period
T_H'	frictionless inlet-bay system Helmholtz period
T_k	period of free mode of oscillation where the subscript indicates the mode of oscillation
t	time
V	mean instantaneous inlet water velocity at a cross section
\bar{V}	mean inlet water velocity at a cross section over a sampling interval
$W_{i,j}$	the grid cell weighting function which is the fraction of total inlet discharge passing through the cell at a time step
x	distance along the longitudinal axis of the inlet
y	distance along a cross section
Δt	time step
$(\tau_{zz})_z$	component of the bottom-stress tensor in the direction of flow

HYDRAULICS OF GREAT LAKES INLETS

by

William N. Seelig and Robert M. Sorensen

I. INTRODUCTION

Numerous bays and harbors are connected to the Great Lakes by jettied inlet channels. These inlet channels are important because they allow (a) access to commercial shipping and recreational boating, (b) migration of fish, and (c) flushing of pollutants from the bays and harbors.

Great Lakes inlet-bay systems are generally smaller than those on the Atlantic, Pacific, and gulf coasts of the United States and respond primarily to the long wave seiche modes of the Great Lakes rather than to the astronomical tides. These seiches have smaller amplitudes and shorter periods than the tides on the ocean coasts.

The major effort of this study involved the collection and analysis of hydraulic data at several inlet-bay systems throughout the Great Lakes during 1974 and 1975. Measurements at Pentwater, Michigan, the primary study location, included simultaneous recording of inlet current velocities and water levels in the bay and in Lake Michigan. At the other locations, only bay water levels were measured. However, hydrographic surveys were obtained for all the inlets investigated, and historic hydraulic data from selected sources were analyzed.

This study defines the hydraulic mechanisms important to Great Lakes inlet-bay systems, develops analytical techniques for the prediction of inlet currents and bay water level oscillations, and presents design data and system response curves for selected inlets.

Field data were analyzed using (a) a formula for estimating the seiche periods of the Great Lakes which are important in producing reversing currents at an inlet; (b) a model that uses bay water level time histories to predict inlet velocities; and (c) a simplified numerical inlet hydraulic model that, when calibrated for friction effects, can be used to predict inlet velocities and bay water level oscillations generated by lake oscillations. These analysis techniques are used with the field data to develop response curves and cumulative inlet current velocity distribution curves for the inlets studied.

II. LAKE AND INLET HYDRAULICS

1. Great Lakes and Inlet-Bay System Hydraulics.

An inlet is a relatively narrow channel which connects a "sea" (or one of the Great Lakes in this study) to a lake or harbor (a "bay" in this study). The bay is large compared to the inlet (i.e., the radius

of the bay is typically larger than the length of the inlet) and the surface area of the bay is much smaller than the surface area of the sea.

a. Causes of Reversing Inlet Currents. Mortimer (1965) and Freeman, Hamblin, and Murty (1974) show that significant reversing inlet current velocities are caused by water level fluctuations in the Great Lakes which generate a hydraulic response in the inlet and bay. The most important Great Lakes water level fluctuations are due to the resonant seiching or oscillation of the particular lake at its fundamental and harmonic periods.

Seiches are initiated by storm pressure and wind forces on the sea which redistribute water in the lake to cause a higher elevation than normal in some areas and lower levels in other areas. When gravity tries to restore the water level, seiches are generated. These seiches usually continue for a number of cycles which may extend over a few days after the storm has passed.

When a seiche is generated in one of the lakes, the water level fluctuations outside an inlet cause a head difference across the inlet, which, in turn, generates a current in the inlet. Water discharge through the inlet results in water added to or removed from the bay so the bay level rises and falls in a pumping fashion for most seiching periods of the lake.

Astronomical tides and other nonseiching long waves cause water level fluctuations of the Great Lakes; however, these fluctuations generally have insufficient amplitudes or are at periods that usually do not significantly influence inlet hydraulics. Storm surge, particularly on shallow Lake Erie, may occasionally generate strong inlet currents.

b. Mathematical Description of Inlet-Bay Hydraulics. The response of an inlet-bay system may be described in terms of the one-dimensional equation of water motion in the inlet and the continuity equation relating the rate of bay level change to the inlet discharge.

The one-dimensional equation of motion along the inlet channel axis can be written:

$$-g \frac{\partial h}{\partial x} = \frac{1}{A_c} \int_{y_1}^{y_2} (\tau_{zx})_z dy + g \frac{\partial h}{\partial x} + \frac{\partial V}{\partial t} \quad (1)$$

where

x = distance along the channel

h = water surface level

A_c = inlet cross-sectional area

V = water velocity in the inlet

y = distance along the cross section

$(\tau_{zx})_z$ = component of the stress tensor in the direction of flow

g = acceleration due to gravity

t = time

The inlet has a width, B , depth, d , length, L , and cross-sectional area, A_c ; the bay has a surface area, A_{bay} . The water levels in the sea and bay are h_s and h_b , respectively (Fig. 1).

Equation (1) equates the horizontal driving force due to the water surface slope with three terms on the right which are the channel frictional resistance, the convective acceleration caused by velocity variation along the channel axis, and the temporal acceleration (or inertia) resulting from velocity variation at a point with time. In nearly prismatic channels, such as many inlets on the Great Lakes, the convective acceleration is often negligible.

The continuity equation, which relates rate of bay water level change, $\partial h_b / \partial t$, to inlet discharge, Q , is:

$$Q = VA_c = A_{bay} \frac{\partial h_b}{\partial t} . \quad (2)$$

A simultaneous solution of equations (1) and (2) for a sinusoidal sea level fluctuation reveals the important response characteristics of an inlet-bay system (Fig. 2). In this figure, the phase lag between the sea and bay water level fluctuations and the amplification of the forcing wave in the bay by the inlet-bay system are plotted as functions of dimensionless period. Dimensionless period is defined as the frictionless inlet-bay system Helmholtz period, T_H' , divided by the forcing wave period, T . The Helmholtz period is that period of the forcing wave which through resonance will cause the largest water level fluctuation in the bay. The bay water level remains approximately horizontal throughout this fluctuation.

The inlet-harbor system response is analogous to the response of a slightly damped spring-mass system or its acoustic counterpart, the Helmholtz resonator. The motion of the mass of water in the inlet channel corresponds to the motion of the mass of the spring-mass system, and the action of gravity on the rising and falling harbor water surface corresponds to the restraining force of the spring.

At values of T_H' / T approaching zero (long wave periods) the water level fluctuations in the bay are the same as those in the sea with no

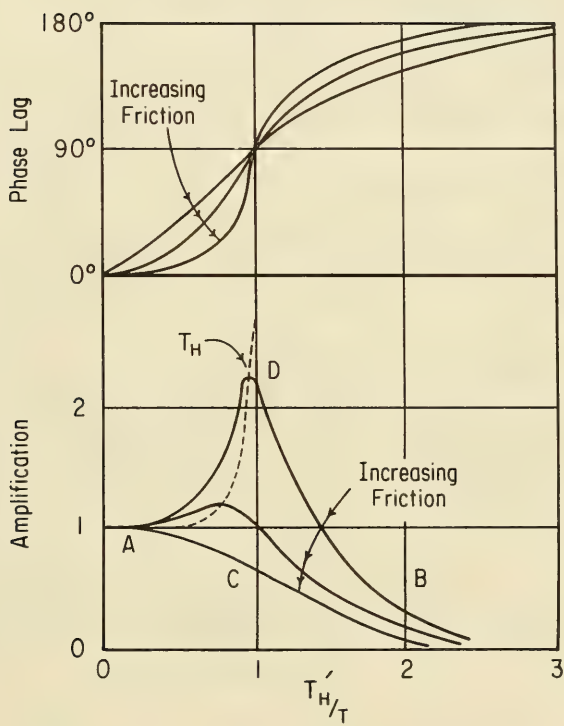


Figure 2. Amplification and phase lag for inlet-bay systems.

phase lag (point A, Fig. 2). At values of T_H'/T approaching 3 (short forcing periods), the inlet-bay system strongly dampens incident waves and water level fluctuations in the sea have little influence on inlet hydraulics (point B, Fig. 2). At values of T_H'/T in the range of $0.25 < (T_H'/T) < 2$, the amount of frictional resistance in the channel has a major influence on the response characteristics. Inlets with high friction, e.g., tidal inlets on ocean coasts, typically have amplifications of less than one. This amplification factor decreases as the forcing wave period becomes shorter (point C, Fig. 2). At most tidal inlets, the primary tidal period is large compared to the inlet-bay Helmholtz period. Low-friction inlets, such as those on the Great Lakes, have amplifications greater than one and phase lags of approximately 90° for forcing waves with $T_H'/T \approx 1$ (point D, Fig. 2) which commonly occur.

2. Prediction of Inlet Velocities.

Prediction of inlet velocities requires (a) the time history of sea or bay water levels, (b) the geometries of the inlet and bay, and (c) a friction-calibrated model to relate water level fluctuations to inlet-bay response.

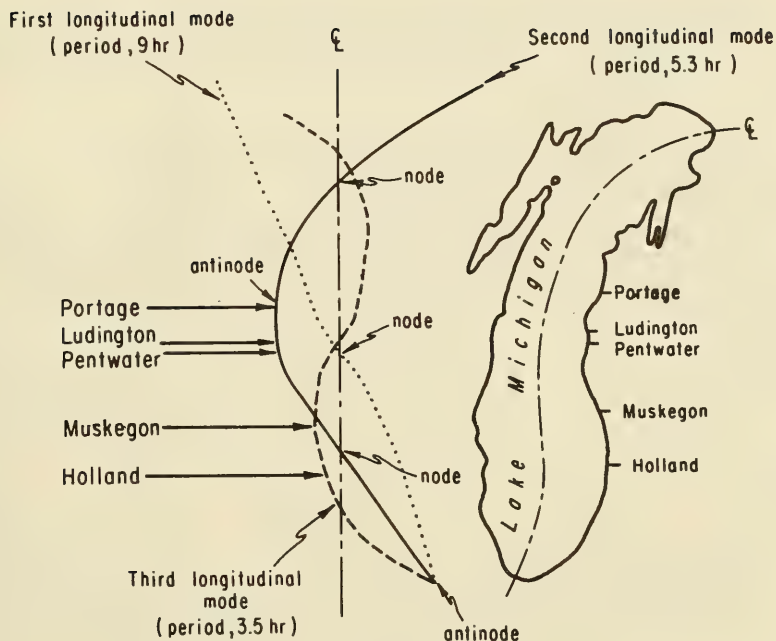
a. Great Lakes Water Level Fluctuations. In general, no methods are presently available for inexpensively predicting all important amplitudes and periods of water level fluctuations at any point in one of the Great Lakes. Therefore, water levels generally must be measured. However, inexpensive schemes are available for accurately predicting some periods and relative amplitudes of seiches of the Great Lakes (Fee, 1968; Rao and Schwab, 1974). Knowledge of the existing wave periods will aid in the design of water level measuring systems, analysis procedures, and preliminary inlet design.

The basic method for estimating one-dimensional fundamental and harmonic seiche periods, T_k , is to determine the time required for a wave to travel twice the length of the basin:

$$T_k = \int_{x=0}^{x=L_b} 2 \frac{dx}{kc}, \quad (3)$$

where k is the mode of oscillation, L_b the length of the basin in the direction of the seiche, and c the speed of the wave. Sample predicted longitudinal seiches for Lake Michigan, using the Fee (1968) computer program, are shown in Figure 3 for modes $k=1, 2$, and 3 .

b. Inlet and Bay Geometries. Hydrographic surveys, including Corps of Engineers dredging records, can be used to determine inlet geometry; i.e., length, width, and depth field. Maps and aerial photos can be used to determine bay surface area.



Relative Seiche Height of Water
(normalized by level at
the southern tip of
Lake Michigan)

Figure 3. Three predicted longitudinal modes of oscillation of Lake Michigan (modified from Mortimer, 1965).

c. Methods of Analysis of Inlet-Bay Hydraulics. Inlet current velocities and bay water surface oscillations may be measured to provide necessary information on the hydraulic characteristics of an inlet-bay system. Techniques used for the field measurements in this study are discussed in Section III, 2.

Several analytical methods are available for predicting inlet hydraulics, depending on the type of information available and required results. Three methods are discussed below.

(1) Estimation of Seiche Periods Important to Inlet Hydraulics.

To estimate which of the Great Lakes seiche periods, T_k , may be important, the frictionless inlet-bay Helmholtz period, T_H' , may be determined from:

$$T_H' = 2\pi \sqrt{\frac{(L+L') A_{bay}}{g A_c}}, \quad (4)$$

where L' is an added channel length determined from:

$$L' = \frac{-B}{\pi} \ln \left[\frac{\pi B}{\sqrt{gd} T_H'} \right], \quad (5)$$

L' accounts for the water masses in motion beyond the ends of the inlet (Miles, 1948). Equations (4) and (5) may be iteratively solved to obtain a value of T_H' . This approach proved to yield reasonably accurate estimates for the inlets considered in this study.

As a first approximation, seiche wave periods which are approximately equal to the frictionless Helmholtz period (e.g., between 0.5 and 2 times T_H' ; see Fig. 2) will probably cause the highest inlet reversing currents.

The seiche node-antinode pattern in the Great Lakes will also influence the importance of the various seiche modes on an inlet-bay system. Seiches with antinodes adjacent to the inlet will produce the largest water level fluctuations. Since ends of the lakes are antinodes for all modes of oscillation along that axis, bays at the end of a lake will normally be subject to higher water level fluctuations than those at other locations; e.g., midway along the longitudinal axis of Lake Michigan, near Pentwater, the first longitudinal mode of oscillation has a node (Fig. 3). Therefore, only small oscillations can be generated in the lake at this point by this mode. The second longitudinal mode of oscillation has an antinode adjacent to Pentwater, so large water level fluctuations in Lake Michigan could be generated outside Pentwater by this mode.

(2) Estimation of Inlet Velocities from Bay Water Level Records.

A method of predicting inlet velocities, if high-quality bay water level

records are available, is to use the continuity equation. Written in finite-difference form, equation (2) becomes:

$$\bar{V} = \frac{A_b \Delta y}{A_c} \left(\frac{h_b - h_b'}{\Delta t} \right), \quad (6)$$

where \bar{V} is the average inlet current velocity at a cross section of area A_c over a water level sampling interval, and Δt , h_b' , and h_b are mean bay water levels at the beginning and end of the sampling interval. Measurements of water level at any point in the bay will be representative of the mean bay level for the Helmholtz mode of oscillation of inlet-bay systems.

This method for predicting inlet velocities is well suited for Great Lakes inlets because inlet and bay geometries are simple and level recorders are easy to install in the protected bays. The sampling interval should be one-twentieth of T_H' or shorter and the stilling well carefully designed for best results (see Sec. III).

(3) A Numerical Model. A relatively simple but extremely useful method of modeling inlet-bay hydraulics is to simultaneously solve the equations of motion and continuity. In this model the inlet channel is divided by a flow net into a grid of subchannels and cross sections. The subscripts i and j describe the location of the cell for subchannels (IC = number of channels) and grid sections (IS = number of sections). The equation of motion for an inlet (eq. 1) rewritten in finite-difference form, and integrated along the axis yields (Seelig, Harris, Herchenroder, in preparation, 1977):

$$\begin{aligned} \frac{dQ}{dt} = & \frac{1}{\sum_{i=1}^{IS-1} \left(\frac{\sum_{j=1}^{IC} L_{ij}/IC}{\sum_{j=1}^{IC} A_{ij}} \right)} \left[\frac{1}{2} \left(\frac{1}{A_s^2} - \frac{1}{A_b^2} \right) + g(h_s - h_b) \right. \\ & \left. - \sum_{i=1}^{IS-1} \frac{1}{\left(\sum_{j=1}^{IC} A_{ij} \right)} \sum_{j=1}^{IC} \frac{g n_{ij}^2 Q}{2.208 D_{ij}^{1/3} A_{ij}^2} \left| \frac{W_{ij} Q}{B_{ij} L_{ij}} \right| \right] \quad (7) \end{aligned}$$

where A_s and A_b are the inlet cross-sectional areas at the sea and bay ends of the inlet, and W_{ij} is a weighting function for distributing flow throughout the inlet. The discharge through a grid cell is equal to the weighting function of the cell, W_{ij} , times the total discharge of the inlet, Q . The Manning's friction factor, n_{ij} , is determined

during calibration of the model (see Seelig, Harris, and Herchenroder, in preparation, 1977).

For M inlets connecting the bay to the sea, the total discharge for all inlets, Q_T , is:

$$Q_T = \sum_{m=1}^M Q_m \quad (8)$$

The continuity equation is written as:

$$\frac{\partial h_b}{\partial t} = \frac{Q_T}{A_{bay}} \quad (9)$$

Bay levels and inlet current velocities are determined by solving the simultaneous differential equations (7) and (9) using a Runge-Kutta-Gill fourth order finite-difference technique in conjunction with initial conditions and the time history of water levels in the sea. Derivation and sample applications of this model are given in Seelig, Harris, and Herchenroder (in preparation, 1977).

To obtain response characteristics similar to Figure 2 for a specific inlet-bay system, the model can be run by assuming sinusoidal seawater level fluctuations with a typical amplitude; e.g., 3 centimeters (0.1 foot) at periods covering the anticipated range of lake oscillation modes. Each run of the model will give predicted bay levels and inlet current velocities for the wave period used. Sample model results for Pentwater inlet are shown in Figure 4. The sea level, predicted bay level, and inlet velocity are shown in the lower part of Figure 4; the importance of each of the terms in the equation of motion, normalized by dividing by the magnitude of the largest term at each time step, is shown in the upper part of the figure. For this condition, the bay level fluctuation is larger than the sea level fluctuation due to inertia in the system and the bay level lags the sea level by 84° (Fig. 4). Plotting results from many runs similar to Figure 4, but with many different forcing periods, will give the response characteristics of the inlet-bay system. These curves for Pentwater (Fig. 5) show that the Helmholtz period with friction, T_H , is 1.8 hours, waves with periods of 1 to 3 hours will be amplified by the system, and waves with a period of 1.4 hours will generate the highest inlet current velocities (3-centimeter wave amplitude assumed). The effect of friction on T_H is demonstrated by the dash-line in Figure 2. The frictionless Helmholtz period is also coincidentally 1.4 hours (from eqs. 4 and 5).

The calculated bay amplification and channel velocity in Figure 5 are for a Manning's $n = 0.036$. The numerical model usually had to be run for three or four cycles for the bay response to build to equilibrium. In the prototype harbor it is likely that equilibrium (full amplification) is never fully achieved. Thus, the calibration curve in Figure 5 forms the upper envelope of measured prototype data.

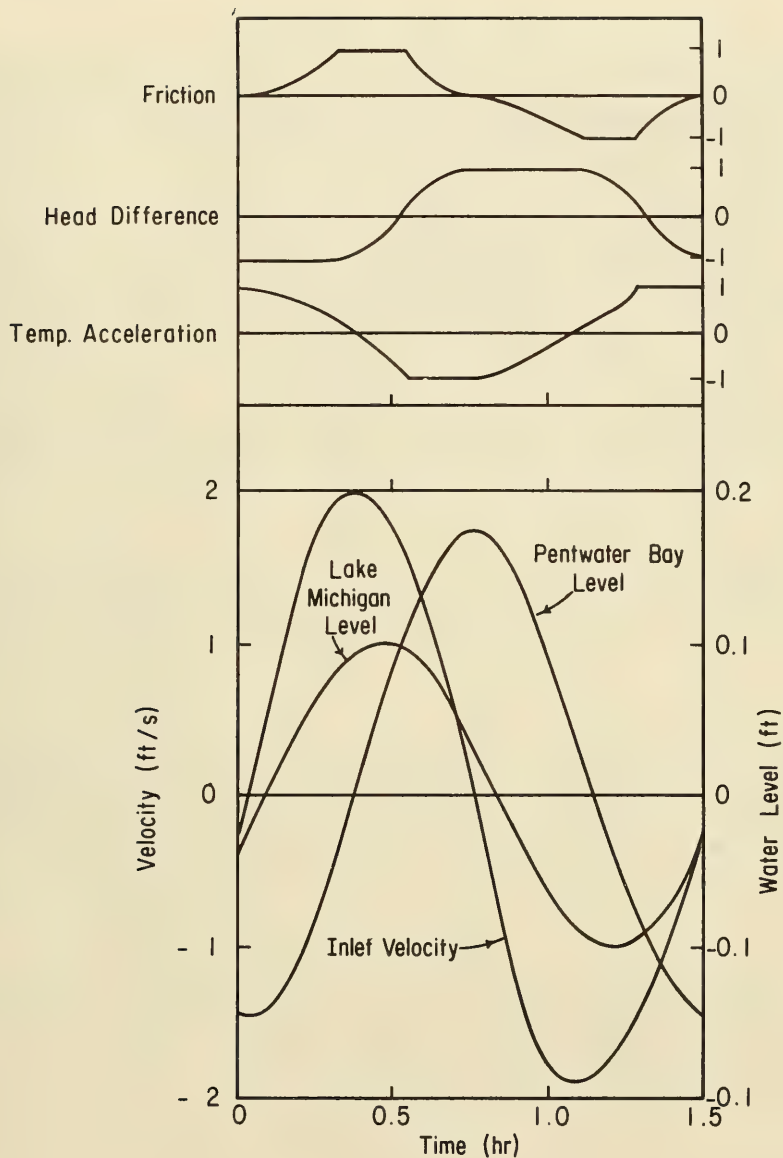


Figure 4. Pentwater response to sinusoidal wave in Lake Michigan (period = 1.5 hours; amplitude = 0.1 foot).

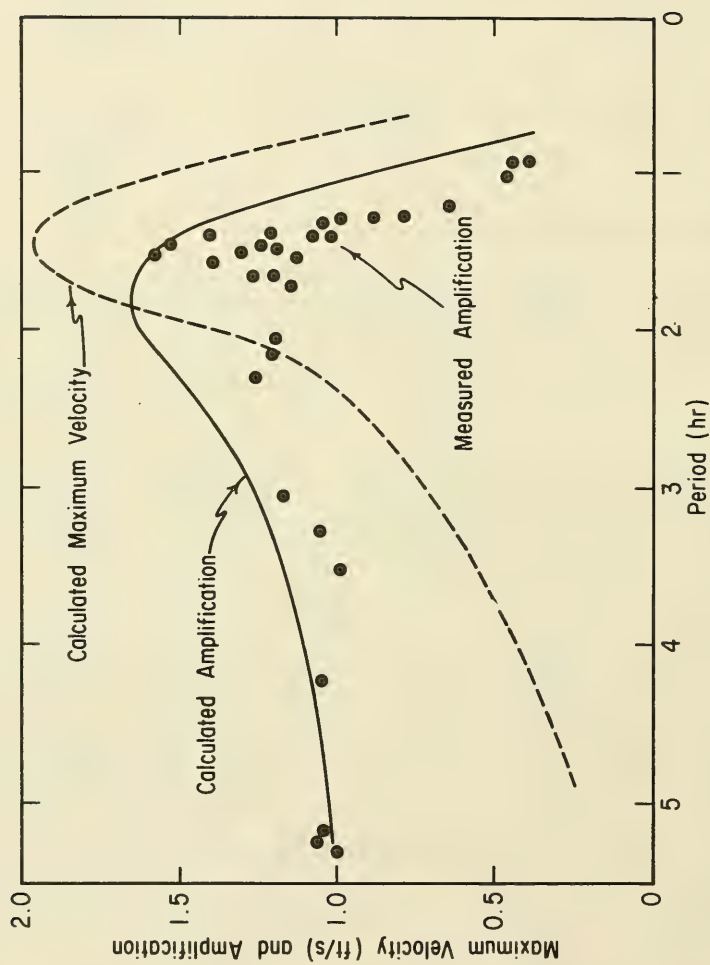


Figure 5. Response to long wave excitation at Pentwater (wave amplitude = 0.1 foot).

If water level fluctuations just outside the inlet are known, the numerical model can be used to predict the resulting channel velocities and bay level fluctuations. Lake Michigan levels just outside of Pentwater were measured by Duane and Saylor (1967) during August 1967 (Fig. 6). Although the record appears confused, a spectral analysis shows that the record is composed of a large number of clearly defined seiche modes of Lake Michigan. Using this record to force the model of Pentwater, a bay level time history is predicted which adequately agrees with measured bay levels (Fig. 6). The inlet-bay system responds primarily to waves with periods of 1 to 3 hours (near the Pentwater Helmholtz period); shorter period waves are damped. This gives the bay level record a smoother appearance than the Lake Michigan record. The maximum predicted inlet current velocity during this episode is 60 centimeters (2 feet) per second.

Other models that neglect temporary acceleration, e.g., Keulegan (1967), should not be used for inlets on the Great Lakes where temporal acceleration, head difference, and friction are important during the response cycle (see Fig. 4).

III. THE FIELD DATA COLLECTION PROGRAM

Measurements were made at a number of inlet-bay systems throughout the Great Lakes during 1974 and 1975 (Fig. 7). These study sites were chosen because the inlets are typical on the Great Lakes, are of special economic importance, or have maintenance problems.

1. Field Measurements.

Pentwater, Michigan, located midway along the longitudinal axis of Lake Michigan, was selected as the primary study location because the inlet-bay system at this location has a simple, fairly common geometry. Good historical field data are also available at Pentwater from Duane and Saylor (1967) who simultaneously measured water levels in Pentwater bay and Lake Michigan as well as inlet velocities during July and August 1967. The U.S. Army Engineer District, Detroit, provided inlet geometry data of the inlet from hydrographic surveys taken twice a year.

Field measurements during this study (Fig. 8) included water level measurements at two locations (east and west ends of Pentwater bay) in 1974, and at one other location in 1975. Current velocities in the inlet were measured concurrently with water levels during both 1974 and 1975. Hydrographic surveys were also taken at Pentwater. Other field data collection sites were: Portage Lake, Ludington, White Lake, Muskegon, and Holland on Lake Michigan (Fig. 8); Little Lake and Duluth-Superior on Lake Superior (Fig. 9); Presque Isle on Lake Erie (Fig. 10); and North Pond and Little Sodus on Lake Ontario (Fig. 11).

Field measurement types, locations, dates, and data sources for all the study sites are summarized in Table 1; approximate dimensions of these inlet-bay systems are summarized in Table 2.

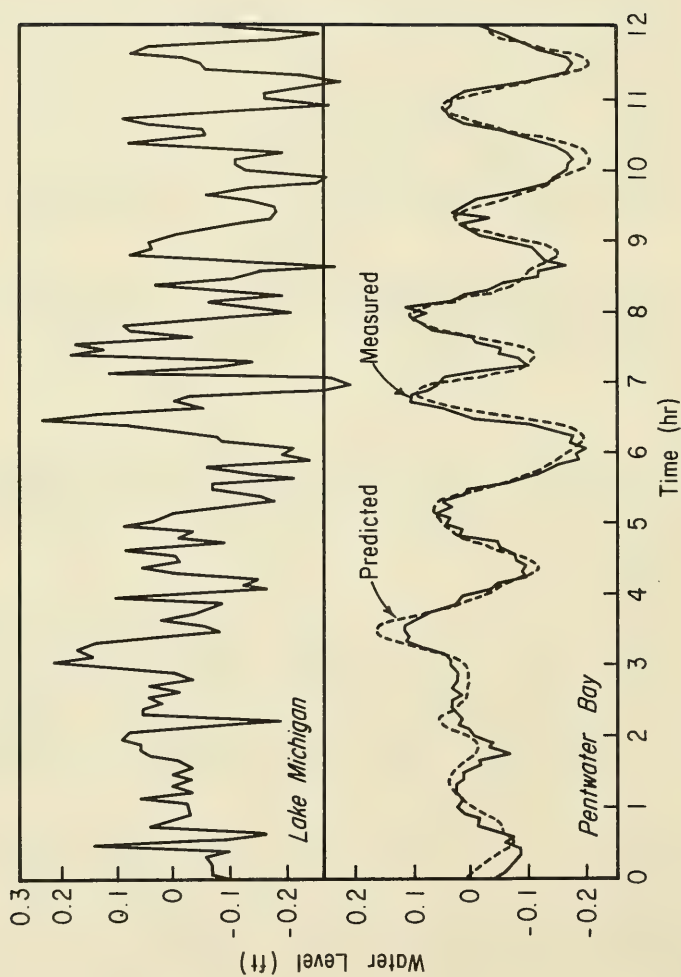


Figure 6. Numerical model water level predictions at Pentwater.

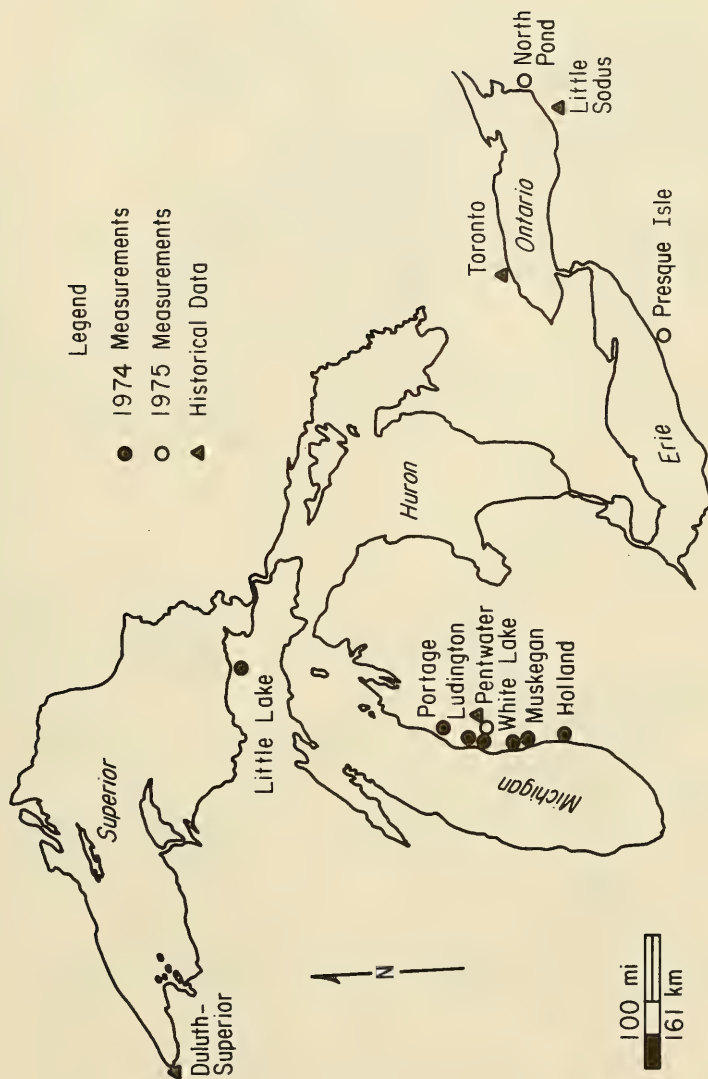


Figure 7. Inlet study sites.

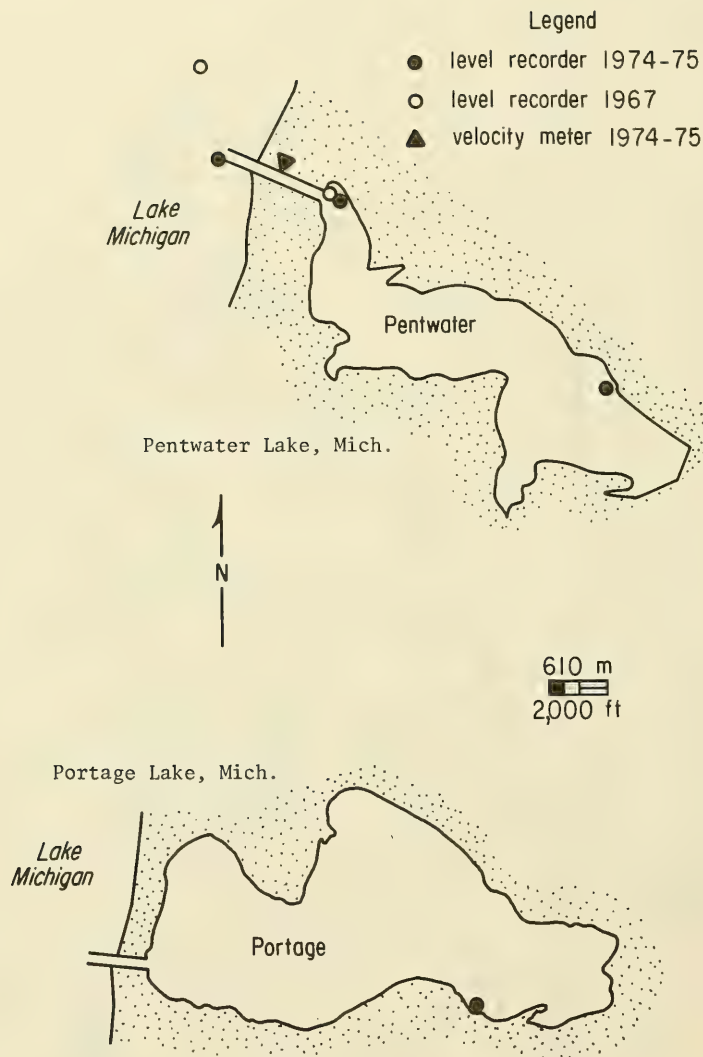


Figure 8. Data collection sites on Lake Michigan.

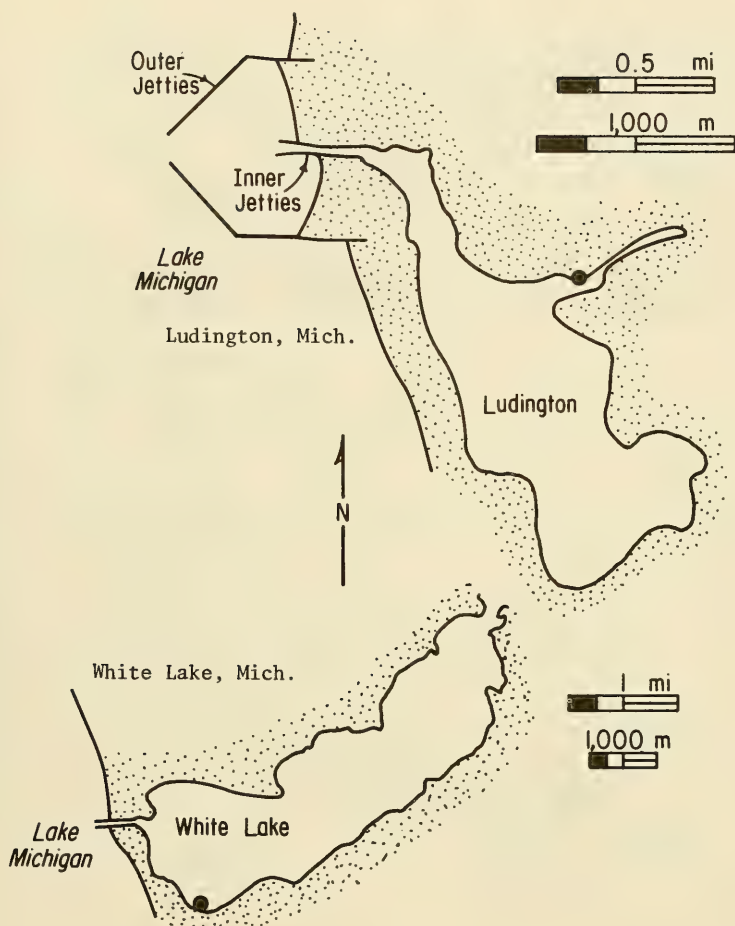


Figure 8. Data collection sites on Lake Michigan--Continued.

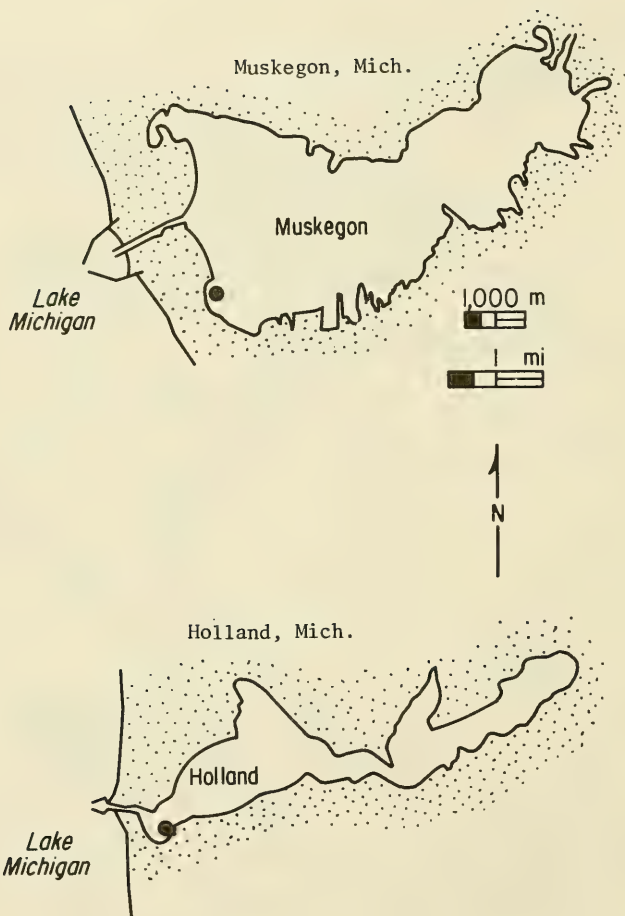


Figure 8. Data collection sites on Lake Michigan--Continued.

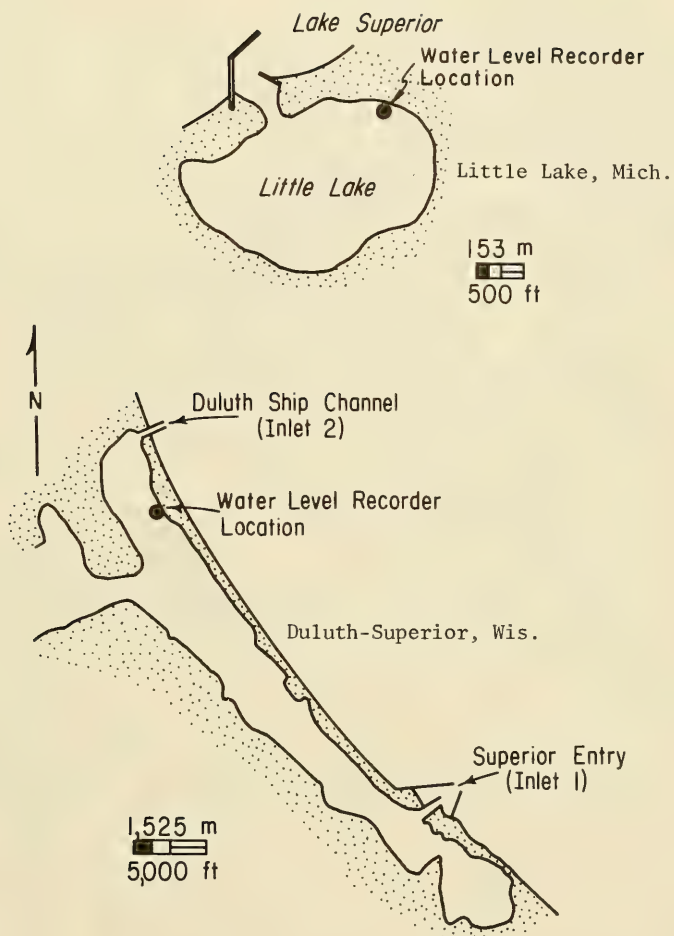


Figure 9. Data collection sites on Lake Superior.

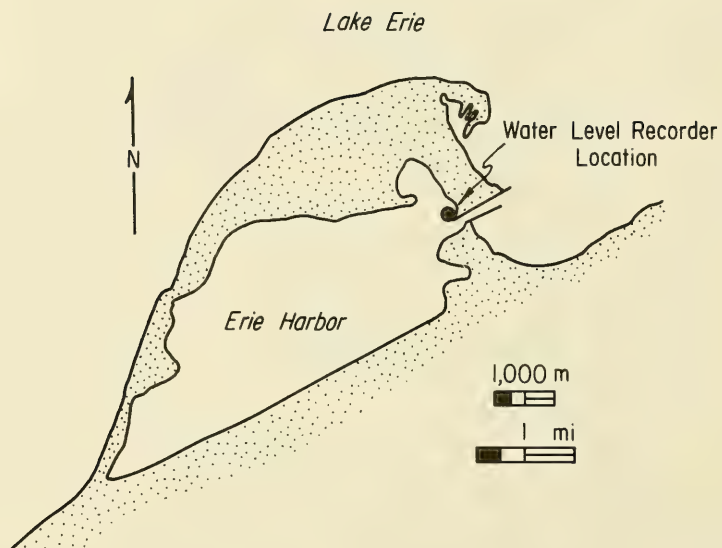


Figure 10. Data collection site on Lake Erie, Presque Isle, Pennsylvania.

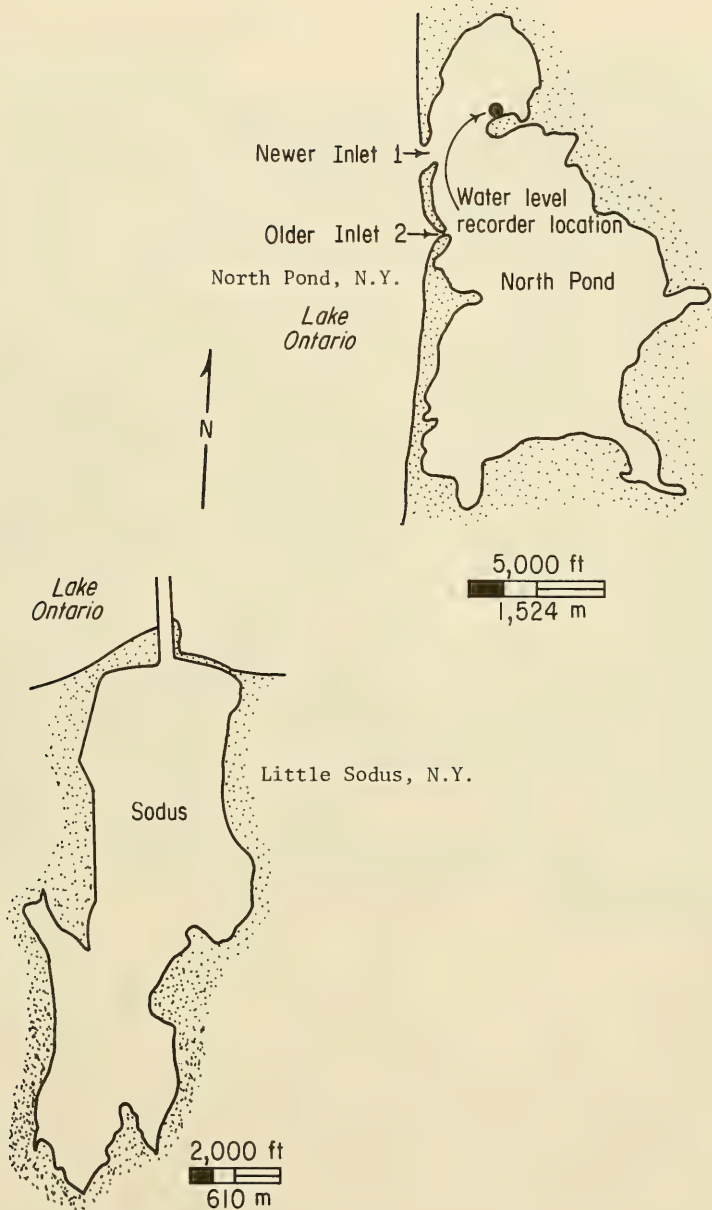


Figure 11. Data collection sites on Lake Ontario.

Table 1. Summary of field measurements.

Location	Water level and dates	Data source	Water level quality	Channel velocities	Inlet geometry (date measured)	Chart No. used to determine bay area ¹
Lake Michigan						
Portage	Portage Lake ² Oct.-Nov. 1974	This study, fall 1974	Good	---	Sept. 1973 ³	777
Ludington	Pere Marq. Lake ²	This study, 1975	Good	---	June 1974 ³	774
Pentwater	Lake Michigan and Pentwater Lake	Duane and Saylor (1967)	Good	Analog record	June 1967 ³	77
	East Pentwater Lake ² Oct.-Nov. 1974	This study, 1975	Good	Analog record	Oct. 1974 ⁴	---
	West Pentwater Lake ² Oct.-Nov. 1974, July-Oct. 1975	This study, fall 1974 This study, 1975	Fair	Analog record	Sept. 1975 ⁴	---
White Lake	White Lake ² Oct.-Nov. 1974	This study, fall 1974	Good	---	May, Aug. 1974 ³	768
Muskegon	Muskegon Lake ² Oct.-Dec. 1974	This study, fall 1974	Good	---	Apr., Aug. 1974 ³	767
Holland	Lake Macataw ² Nov. 1974 (analog)	National Oceanic and Atmospheric Administration records	Poor	---	Mar. 1974 ³	763
Lake Superior						
Little Lake	Little Lake ⁵ May-Aug. 1975	This study, 1975	Fair	---	June 1975 ³	92
Duluth-Superior	Duluth Harbor ² June 1973 (analog)	National Oceanic and Atmospheric Administration records	Fair	Analog record gages had not recently been calibrated ³	Oct. 1974 ³	966
Lake Erie						
Presque Isle	Harbor levels ² May-Oct. 1975	This study, 1975	Good	---	Sept. 1975 ³	332
Lake Ontario						
Little Sodus	Bay levels ² May-Nov. 1972	National Oceanic and Atmospheric Administration records	Poor	---	May 1973 ³	22
North Pond	Bay levels ² May-Oct. 1975	This study, 1975	Poor	---	1975 several surveys made by University of Buffalo under contract	22

¹National Oceanic and Atmospheric Administration, Lake Survey Center.²A 5-minute sampling interval.³Corns of Engineers records.⁴This study, fall 1974.⁵A 2-minute sampling interval.

Table 2. Inlet and bay geometric measurements.

Inlet-bay system	Date	Inlet length (ft)	Minimum inlet cross-sectional area (ft ²)	Inlet width at minimum area (ft)	Surface area of bay (ft ² × 10 ⁶)
Portage	Sept. 1973	2,090	4,820	355	86.1
Ludington	June 1974	3,420	5,640	185	19.0
Pentwater	June 1967	2,025	1,700	145	18.1
Pentwater	4 Oct. 1974	2,025	1,868	146	18.1
White Lake	May-Aug. 1974	1,720	2,978	193	94.6
Muskegon	Apr.-Aug. 1974	6,425	7,450	285	182.0
Holland	Mar. 1974	2,505	4,300	180	77.6
Little Lake	June 1975	1,260	1,850	295	4.2
Duluth-Superior (inlet 1)	Oct. 1974	3,700	14,500	503	160.0
(inlet 2)		1,600	8,850	300	
Presque Isle	Sept. 1975	5,290	13,130	365	14.8
Little Sodus	May 1973	1,800	4,340	260	32.2
North Pond (inlet 1)	Aug. 1975	770	1,050	440	93.0
(inlet 2)		960	234	230	

2. Equipment.

Small-amplitude sea level fluctuations with a period of approximately the Helmholtz period of the inlet-bay system may generate relatively high inlet water velocities as shown in numerical model calculations (see Sec. II). Therefore, a water level recording system must be carefully designed for each location to measure small amplitude, but potentially important long waves. At the same time, this recording system should eliminate any short-period, large-amplitude noise (e.g., wind waves) that may mask the long waves in the record. For example, at Pentwater, records should measure the low-amplitude waves with a period of 1 hour or longer, and should exclude wind waves and other noise with periods of 1 minute or less.

One method of designing a stilling well to meet these requirements is to use the linear damping well design (Noye, 1974). This stilling well consists of a vertical cylinder with a sealed bottom and open to the lake through a long, thin tube. Friction in the tube and the relative cross-sectional areas of the tube and stilling well cause the system to respond directly to long waves outside the well and to drastically dampen the short-period noise. Design of stilling wells is discussed by Seelig (1977).

Fisher-Porter series 1500 digital float-type water level recorders with a vertical resolution of 3 millimeters (0.01 foot) and sampling intervals of 2 or 5 minutes were used to measure water levels in the stilling wells. Data were collected on punched tape. Water levels were measured for several months at each location (see Table 1).

Inlet velocities at Pentwater were measured during 1974 and 1975 using a Bendix current meter suspended by a cable approximately midway along the channel, 4.5 meters (15 feet) from the north wall at mid-depth. Velocity data were recorded on a strip chart and later digitized for analysis at the same time interval as the water level data.

3. Data Reduction and Analysis Techniques.

Initially, the Helmholtz period of each inlet-bay system and the free seiching mode periods were calculated for the lakes and bays surveyed in this study (see procedures in Sec. II). These calculations, in conjunction with a survey of published data on Great Lakes resonance characteristics, gave an indication of the period and magnitude of important long waves that could be expected at each location. The information was used in the design of water level measurement equipment (discussed in previous section).

When the field data collection program was completed, the digital punched-tape water level records were mechanically converted to punchcards for computer analysis. The first procedure for studying these data included plotting the records for visual inspection. Then, a fast Fourier

transform and cosine bell function were used to obtain a spectral analysis of each record (Harris, 1974). A record length of 512 points was used in these analyses to obtain detailed spectral line resolution. Spectral analysis indicated the period and amplitude of long waves of interest (0.5 to 5 hours) in the record. This analysis is necessary because several long waves are generally simultaneously present and the superposition of the waves gives the impression of a confused record. Examples of spectra for Pentwater bay water levels recorded during 5 to 14 May 1975 are shown in Figure 12.

If water level records were of good quality, levels measured in the bay were then used to predict inlet water velocities using the finite-difference continuity equation (6).

The resolution of the continuity equation should be checked to judge the usefulness of the predicted velocities; e.g., at Pentwater, the level recorder has a vertical resolution of 3 millimeters, the sampling interval is 5 minutes (300 seconds), and the ratio $A_{bay}/A_c = 10^4$, so the velocity prediction resolution, V_p , based on equation (6), is:

$$V_p = 10^4 \left(\frac{0.01}{300} \right) = 0.33 \text{ foot or } 10.1 \text{ centimeters per second.} \quad (10)$$

Thus, the velocity will be expressed as multiples of 10.1 centimeters (0.33 foot) per second which may be adequate for many purposes. For example, if A_{bay}/A_c was 10^5 , then with the given vertical resolution, velocities could only be expressed as multiples of 100 centimeters (3 feet) per second, which is inadequate for most purposes.

At Pentwater, the measured velocities in the inlet were digitized at a sampling rate of 5 minutes so that a direct comparison could be made of measured and predicted (eq. 6) velocities. Cumulative frequency distributions of measured and predicted (eq. 6) inlet velocities are shown in Figure 13. The 2 months of record in 1974 show that velocities predicted by continuity are slightly higher than measured velocities, but adequate for many design purposes.

IV. RESULTS

1. Seiching of the Great Lakes.

Free modes of oscillation of the Great Lakes have been identified by spectral analysis of water level records in this study and others (Mortimer, 1965; Mortimer and Fee, 1974; Hamblin, 1975; Rao and Schwab, 1974), and have been predicted using numerical techniques (Rockwell, 1966; Mortimer, 1965; Birchfield and Murty, 1974; Rao and Schwab, 1974). Table 3 summarizes the known modes of oscillation of Lakes Michigan, Superior, Erie, and Ontario.

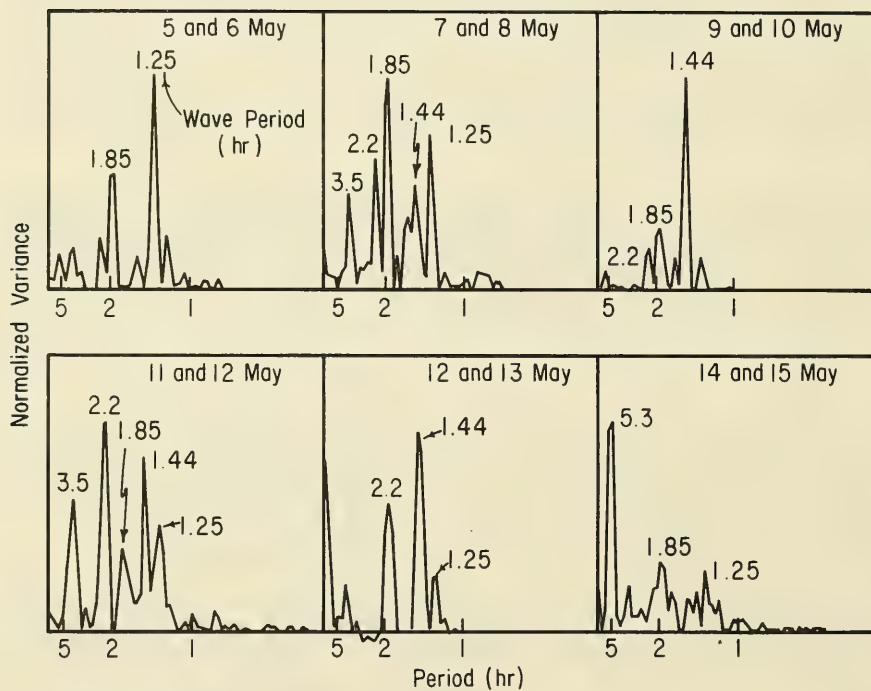


Figure 12. Sample spectra of Pentwater bay water levels (May 1975).

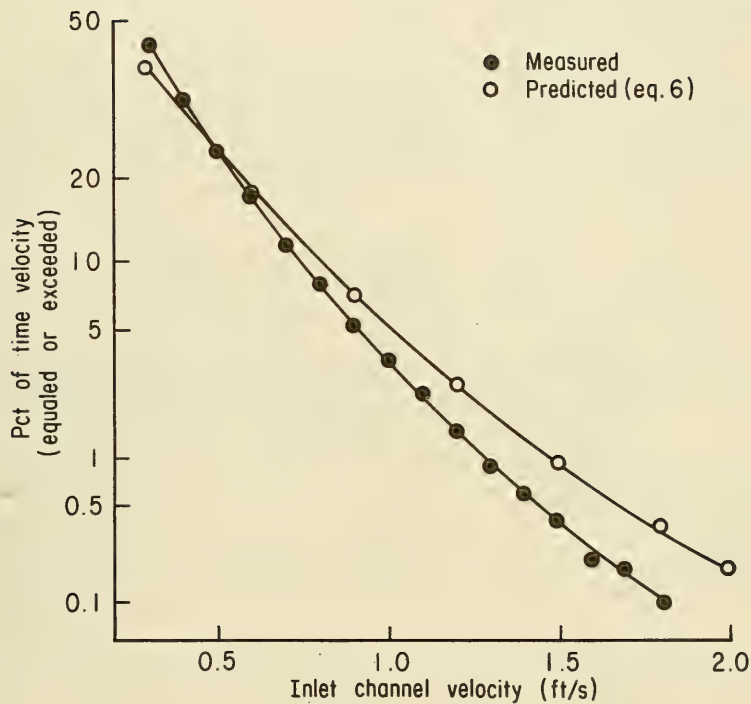


Figure 13. Measured and predicted inlet velocity cumulative frequency distributions at Pentwater, Michigan (October-November 1974).

Table 3. Modes of oscillation of the Great Lakes¹.

Mode ²	Period (hr)			
	Michigan	Superior	Erie	Ontario
1L	9.0	7.8	14.4	4.9
2L	5.3	4.8	9.1	2.9
3L	3.5	3.6	5.8	2.4
4L	3.0	?	4.1	1.6
5L	2.5	3.0	3.6	1.37
6L	1.85	2.2	3.0	1.25
7L	1.58	1.7	2.5	0.9
8L	1.44	1.5	2.2	0.85
9L	1.25	1.2	2.0	---- ³
10L	----	1.14	1.8	----
1T	2.2			
2T	1.1			
?	0.97	0.95	1.2	
?	0.85	0.66 to 0.68	0.92	
?	0.65	0.58	0.69	
?		0.52 to 0.53	0.54	
?		0.49		

¹Observed, computed by Fee (1968) program and Rockwell (1966) data, and compiled from many sources.

²L = longitudinal, T = transverse; ? = other observed periods, mode unknown. Two-dimensional modes are not considered.

³Value unknown.

2. Predicted Inlet-Bay Response to Monochromatic Long Wave Forcing.

The response of each of the inlet-bay systems (Table 2) to uniform long wave forcing was evaluated using the numerical model. In this analysis, a sinusoidal wave was used to force the model for several cycles (generally four) until the inlet-bay system response became periodic. The results will give an upper limit of wave amplification and inlet velocities because the prototype will generally not reach a periodic condition.

Each inlet was modeled by using a grid system with one to three channels and two to seven cross sections. The complexity of the inlet determines the number of grids used to model the friction; e.g., Pentwater with a constant width and only slight changes in depth along the length of the inlet, was modeled using one channel and four cross sections. Little Lake, with a more irregular inlet, was modeled using three channels and five cross sections.

Pentwater, Michigan, and Toronto, Canada (Freeman, Hamblin, and Murty, 1974), are the two harbor systems on the Great Lakes with water levels recorded simultaneously inside and outside the harbor to provide the necessary information for calibration of the numerical model. These models were calibrated by varying the value of the Manning's friction factor, n , so that the model long wave amplification is a best-fit upper envelope of prototype measurements. Values of n of 0.036 and 0.062 were found for Pentwater and Toronto, respectively (Figs. 14 and 15).

The numerical model as used in these analyses did not explicitly account for energy losses due to radiation of long waves into the sea or entrance and exit losses. These losses are incorporated into the model in the form of bottom friction through model calibration. Including these losses in the friction term means that Manning's n calibrated for Great Lakes inlets is higher than that used in open channel flow computations.

In the numerical model, the magnitude of Manning's n determines the amount of energy dissipated. Larger values of n will cause higher amounts of energy loss which results in less wave amplification in the bay and lower inlet velocities. The influence of n on inlet-bay response to long wave forcing at Pentwater is shown in Table 4.

Table 4. Influence of Manning's n on inlet-bay response at Pentwater.

n	$(a_b/a_o)_{max}$	V_{max} , (ft/s for $a_o = 0.1$ ft)
0.26	2.1	2.8
0.36	1.7	1.9
0.46	1.4	1.5

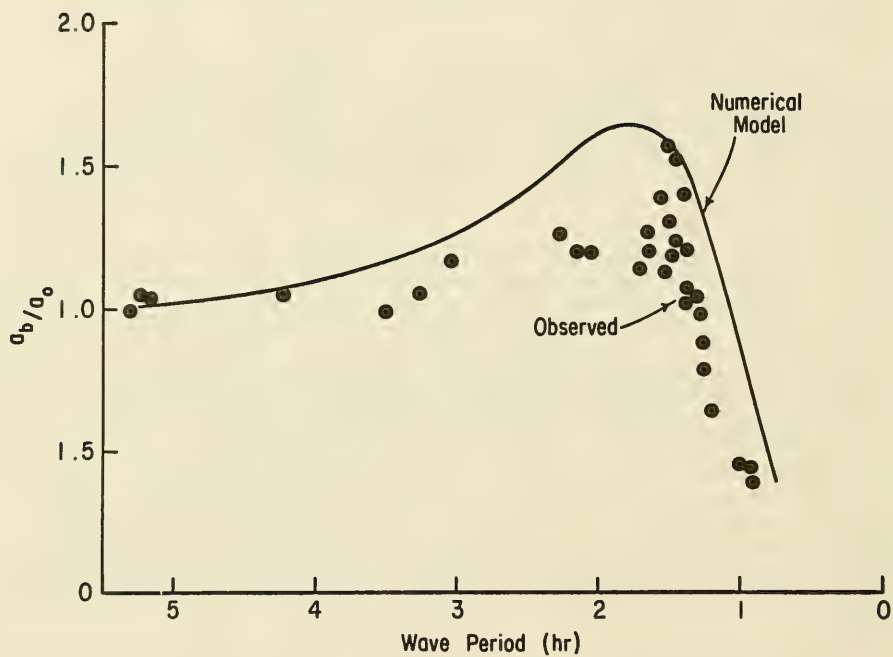


Figure 14. Pentwater response to long wave forcing ($n = 0.036$).

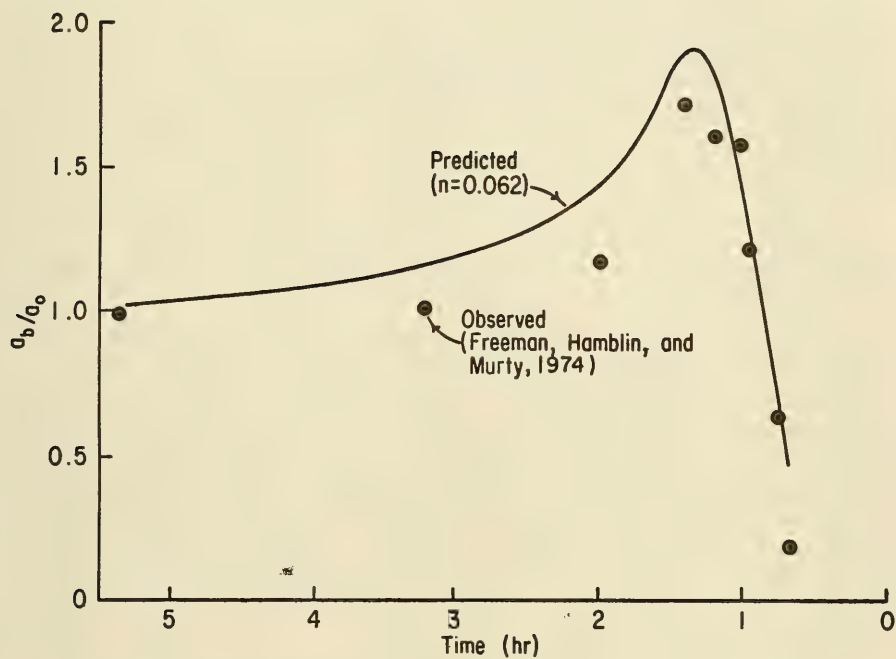


Figure 15. Toronto response to long wave forcing.

Values of n were estimated for other Great Lakes inlets, based on experience at Pentwater and Toronto. Then, the frictionless Helmholtz period was estimated for each inlet-bay system using equations (4) and (5), and the numerical model. Numerical model and frictionless Helmholtz period results are summarized in Table 5.

The amplitude response curves and predicted maximum velocities are shown for selected inlets on Lake Michigan (Fig. 16), Lake Superior (Fig. 17), and Lakes Erie and Ontario (Fig. 18). A 3-centimeter monochromatic forcing-wave amplitude was used in these models. For waves of different amplitudes, the maximum inlet velocity is approximately proportional to amplitude. Water level changes throughout the forcing cycle cause nonlinear effects (i.e., a_b/a_o is slightly different at high and low water), so that the mean of ebb and flood conditions is used in the response curves in this study.

This analysis shows that all of the jettied inlet systems studied have significant inertial effects because long waves at or near the Helmholtz period of each system have higher amplitudes in the bay than in the Great Lakes.

The inlet-bay systems modeled have a wide variation in response characteristics from one system to another because of the complicated interactions between the four terms in the equation of motion of the inlet and the response of the bay to the inlet. Pentwater, for example, has a moderate amount of wave amplification and produces inlet velocities greater than 30 centimeters per second (1 foot per second) for forcing waves of 3-centimeter amplitude and periods ranging from 0.9 to 2.5 hours (Fig. 16). White Lake has less wave amplification, but the interaction between the inlet and bay produces higher velocities over a wider range of forcing periods (greater than 30 centimeters per second for periods of 1 to 5.6 hours) (Fig. 16). Since Little Lake and Presque Isle have the capacity to generate reversing currents in only a narrow window of forcing periods, it is unlikely that significant reversing currents will be frequently generated (Figs. 17 and 18).

Duluth-Superior has the highest capacity for generating reversing currents for a given wave amplitude with maximum velocities occurring at a theoretical forcing period of 1.1 hours. The mean velocities in Duluth (inlet 2), are approximately 1.5 times larger than in Superior (inlet 1) (Fig. 17). A unique feature of the Duluth-Superior system is that the model predicts a net flow into the harbor through the Duluth inlet and a net outflow through the Superior inlet when the forcing period is near 1 hour (Table 6). This asymmetry in flow throughout the forcing cycle will generate a small counterclockwise net flow throughout the inlet-bay system at Duluth-Superior.

North Pond, in 1975, had two short natural inlets connecting a relatively large bay to Lake Ontario. North Pond does not amplify long waves because the mass of water in the inlets is small compared to bay size, and friction in the inlets is high due to the shallow-water depths (Fig. 18). Since friction is high, North Pond behaves like a traditional tidal inlet with a balance between head and friction in the inlets.

Table 5. Predicted periods of maximum wave amplification and maximum inlet velocities¹.

Location	Manning's n	Helmholtz period (hr)		Period of maximum velocity (hr) (numerical model)
		Frictionless eqs. (4) and (5)	Numerical model	
Lake Michigan				
Portage	0.036	2.1	2.9	1.8
Ludington	0.062	1.2	1.0	0.9
Pentwater	0.036 ²	1.4	1.8	1.4
White Lake	0.036	2.6	4.5	2.3
Muskegon	0.062	3.8	5.0	3.1
Holland	0.045	2.2	2.5	1.7
Lake Superior				
Little Lake	0.036	0.6	0.6	0.5
Duluth-Superior	0.062	1.4	1.3	1.1
Lake Erie				
Presque Isle	0.062	0.8	0.6	0.5
Lake Ontario				
Little Sodus	0.036	1.4	3.3	2.7
North Pond	0.036	3.0	---3	---
Toronto	0.062 ²	1.2	1.3	1.1

¹Using a monochromatic wave with amplitude, $a_0 = 0.1$ foot (3 centimeters).

²Calibrated.

³Friction is dominant so that amplification does not occur.

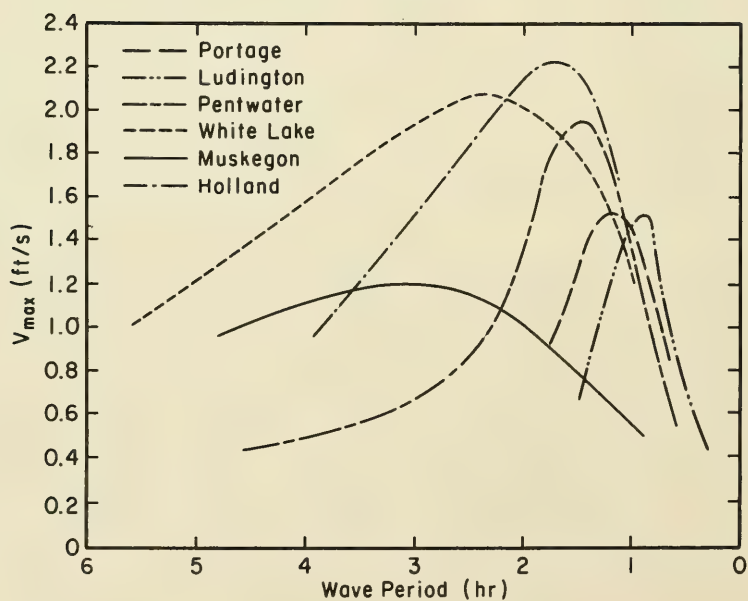
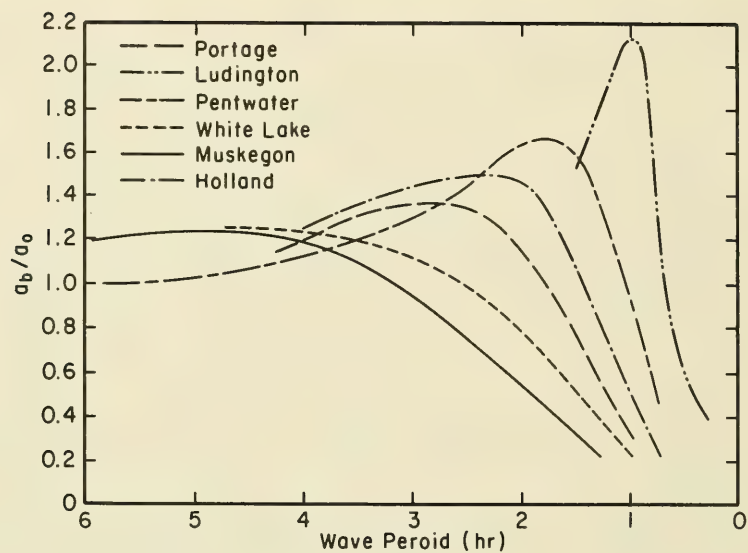


Figure 16. Predicted response of inlet-bay systems on Lake Michigan to monochromatic forcing ($a_0 = 0.1$ foot).

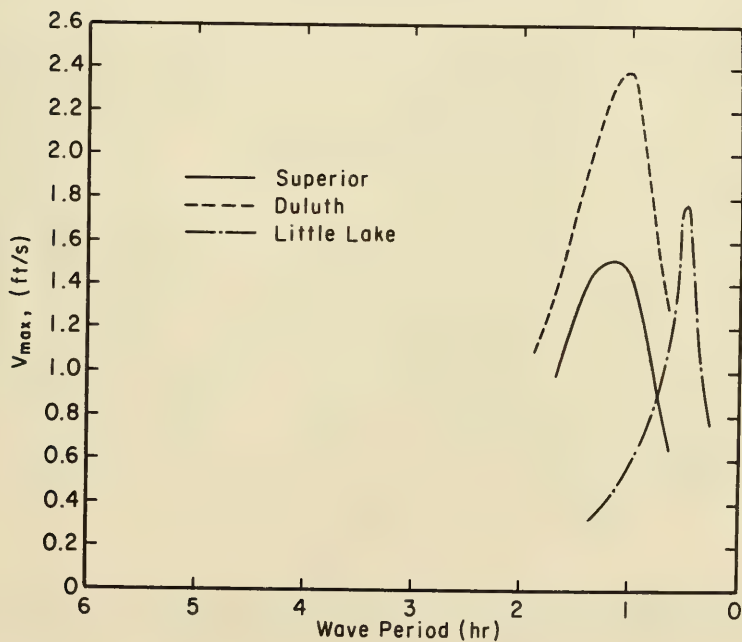
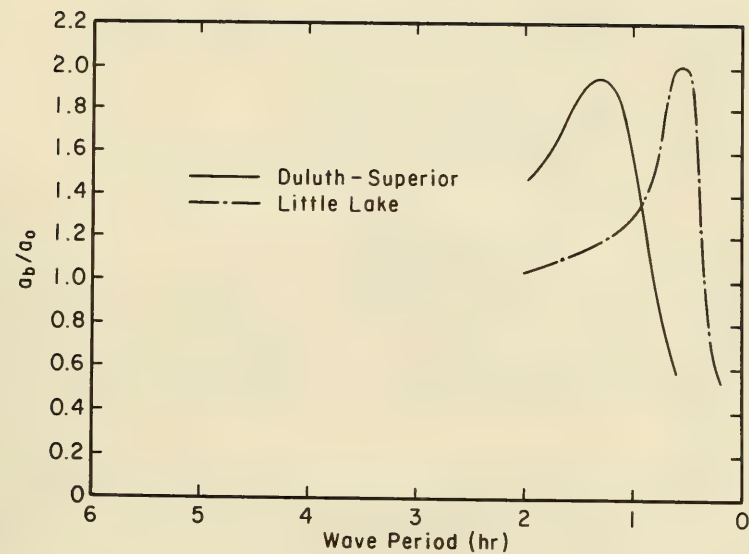


Figure 17. Predicted response of inlet-bay systems on Lake Superior to monochromatic forcing ($a_0 = 0.1$ foot).

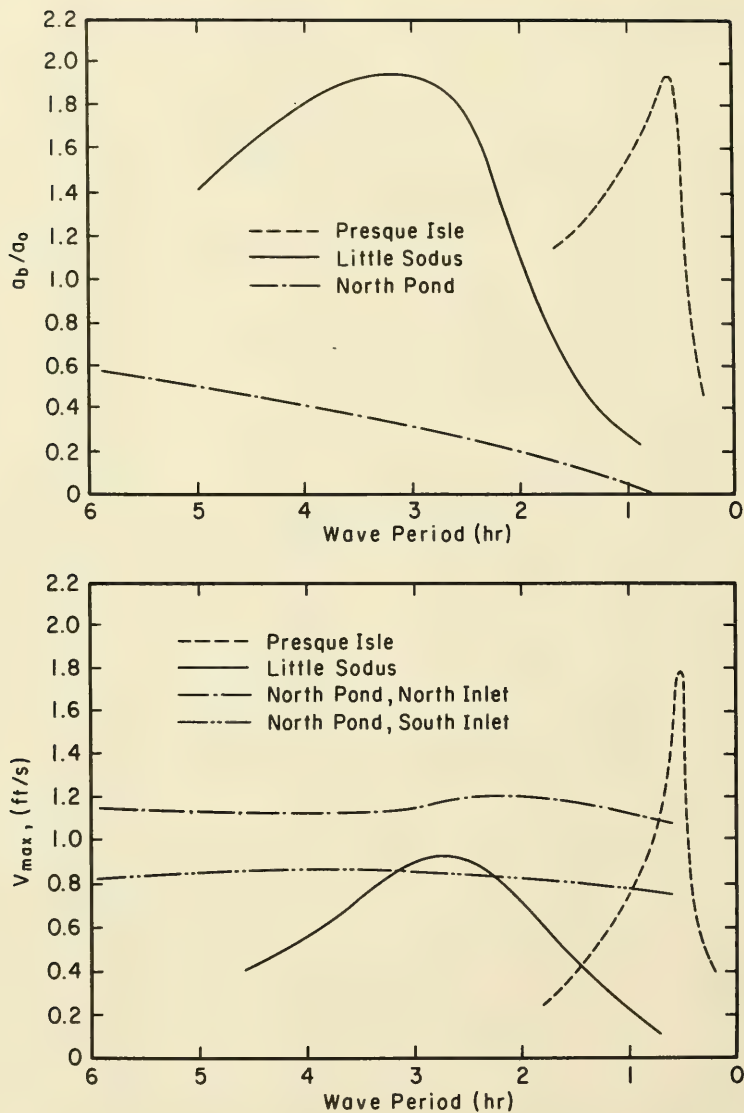


Figure 18. Predicted response of inlet-bay systems on Lakes Erie and Ontario to monochromatic forcing ($a_0 = 0.1$ foot).

Table 6. Predicted Duluth-Superior maximum inlet water velocities for a forcing wave of 1 hour ($a_0 = 3$ centimeters).

	Velocities, cm/s (ft/s)	
	Duluth	Superior
Flood	73 (2.4)	43 (1.4)
Ebb	64 (2.1)	49 (1.6)

A unique feature of the North Pond inlets is that the maximum velocity in the inlets is predicted to be approximately the same over a wide range of forcing periods because of the approximately linear relation between wave amplitude propagation in the bay and wave period (Fig. 18). The velocities in the northern inlet at North Pond, the most recently formed inlet, are predicted to be 1.4 times larger than those in the older inlet.

The numerical model for Pentwater was run using monochromatic forcing waves with the various modal periods of oscillation of Lake Michigan (Table 3). The predicted amplification and maximum velocity for a forcing wave of $a_0 = 0.1$ foot are listed in Table 7. From this analysis, the sixth through ninth longitudinal modes of oscillation of Lake Michigan are predicted to cause the largest wave amplification and generate the highest relative velocities. However, analysis of the node-antinode pattern of Lake Michigan shows that only even modes of oscillation will have antinodes, and cause significant water level fluctuations adjacent to Pentwater (Fig. 3). Since odd modes of oscillation have a node near Pentwater (Fig. 3), even the presence of one or more of the odd modes of oscillation in Lake Michigan will cause only small water level fluctuations near Pentwater. This means that the sixth and eighth longitudinal modes of oscillation of Lake Michigan will probably have the largest influence on the hydraulics of Pentwater.

3. Observed Lake Level Fluctuations, Bay Response, and Inlet Velocities.

The first obvious characteristic of Great Lakes water level fluctuations is that they are not uniform (as assumed in the previous section). Therefore, the monochromatic analysis can be used to obtain an upper estimate of bay wave amplification and inlet velocities; however, a complete analysis is necessary for an accurate estimate of response to a particular Great Lakes water level time history. Sample water level records from Lake Michigan and Pentwater bay, along with spectral analysis of 42-hour records to show typical bay response, are plotted in Figures 19, 20 and 21.

Figure 19 shows a storm event on Lake Michigan when several modes of oscillation were excited by meteorological effects. The second longitudinal mode of Lake Michigan (5.3 hours) and a 0.65-hour wave are particularly dominant. All of the 5.3-hour wave propagates into Pentwater

Table 7. Numerical model prediction of Pentwater response to Lake Michigan modes of oscillation.

Mode	T_n , (hr)	a_b/a_o	V_{max} for $a_o = 0.1$ ft (ft/s)
x 1L	9.0	1.0	0.0
2L	5.3	1.0	0.2
x 3L	3.5	1.2	0.5
4L	3.0	1.3	0.7
x 5L	2.5	1.4	0.9
1T	2.2	1.6	1.1
6L	1.85	1.7	1.6
x 7L	1.58	1.6	1.9
8L	1.44	1.5	2.0
x 9L	1.25	1.3	1.9
?	0.97	0.8	1.5
?	0.85	0.6	1.1

Notes--L = longitudinal modes of oscillation of Lake Michigan.

T = transverse modes of oscillation of Lake Michigan.

? = observed oscillation, mode unknown.

x = wave has a node near Pentwater.

Modes 6L to 9L have modes of oscillation with large a_b/a_o and V_{max} for Pentwater.

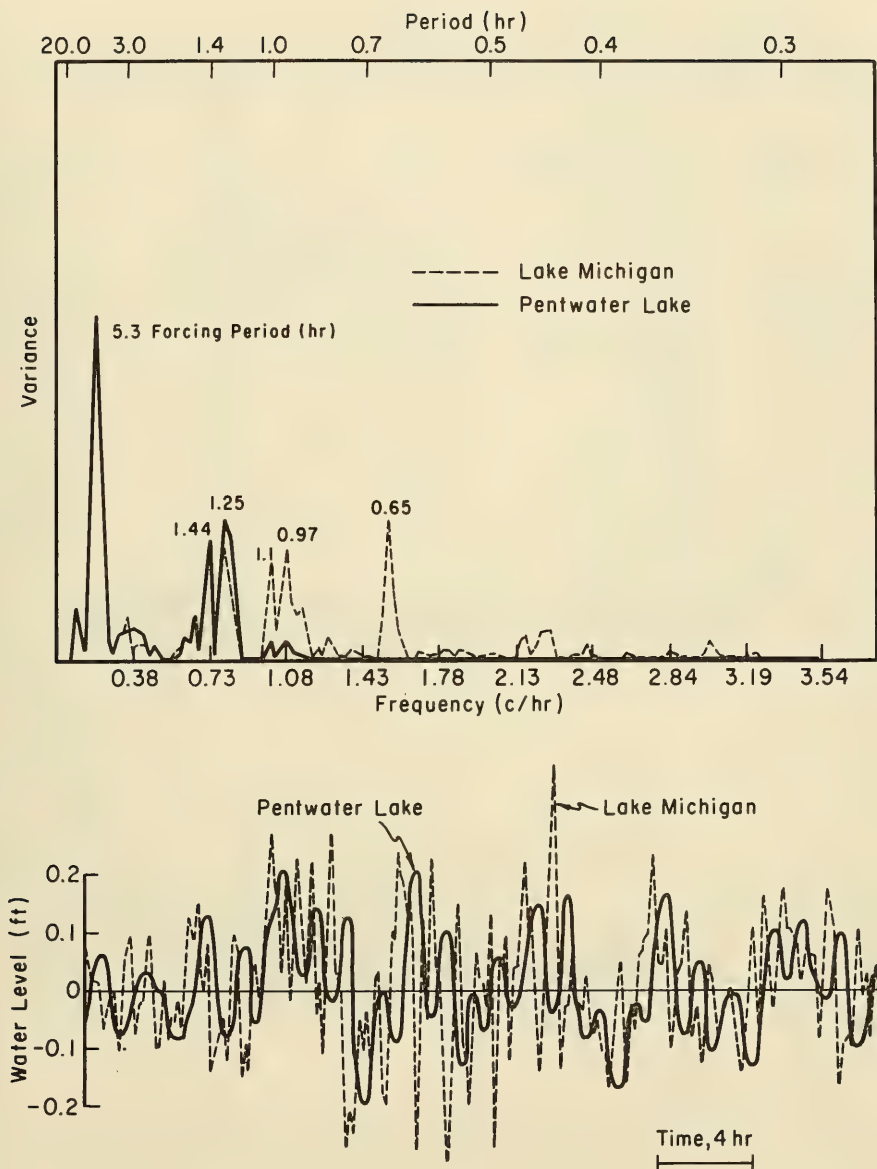


Figure 19. Sample Pentwater and Lake Michigan water levels and spectra.

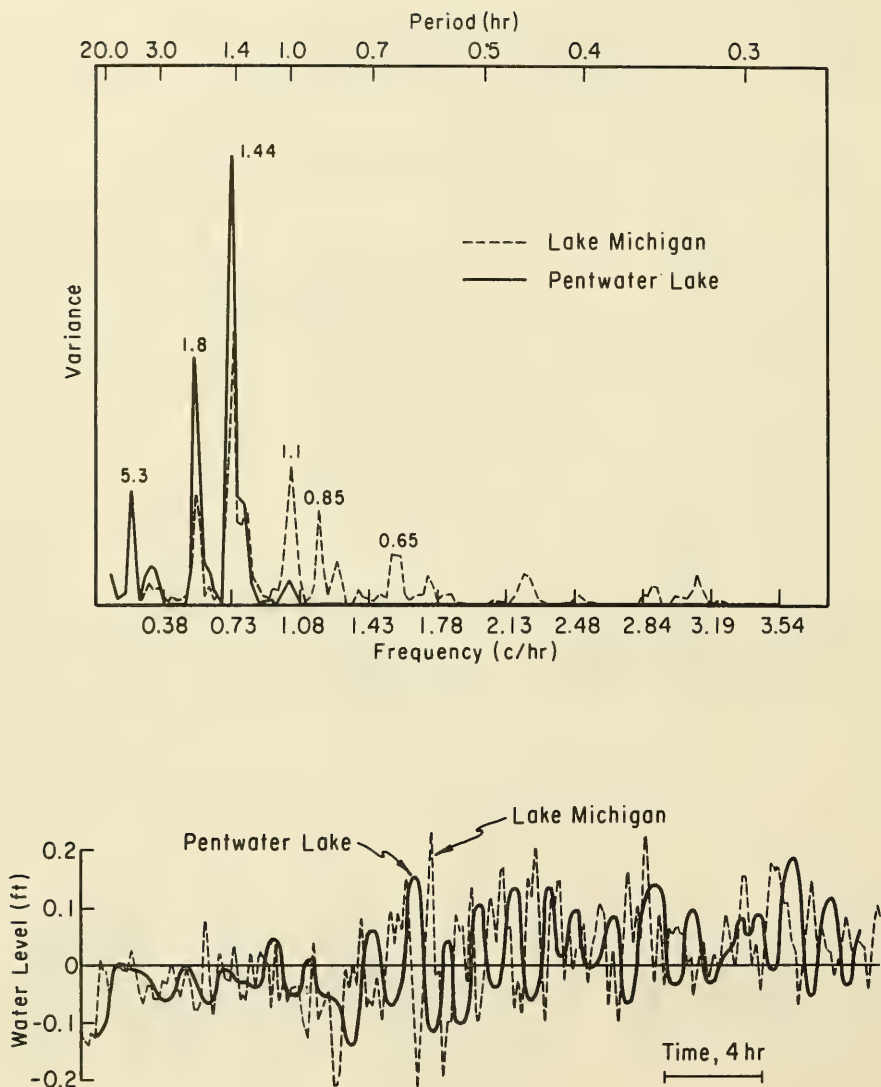


Figure 20. Sample Pentwater and Lake Michigan water levels and spectra.

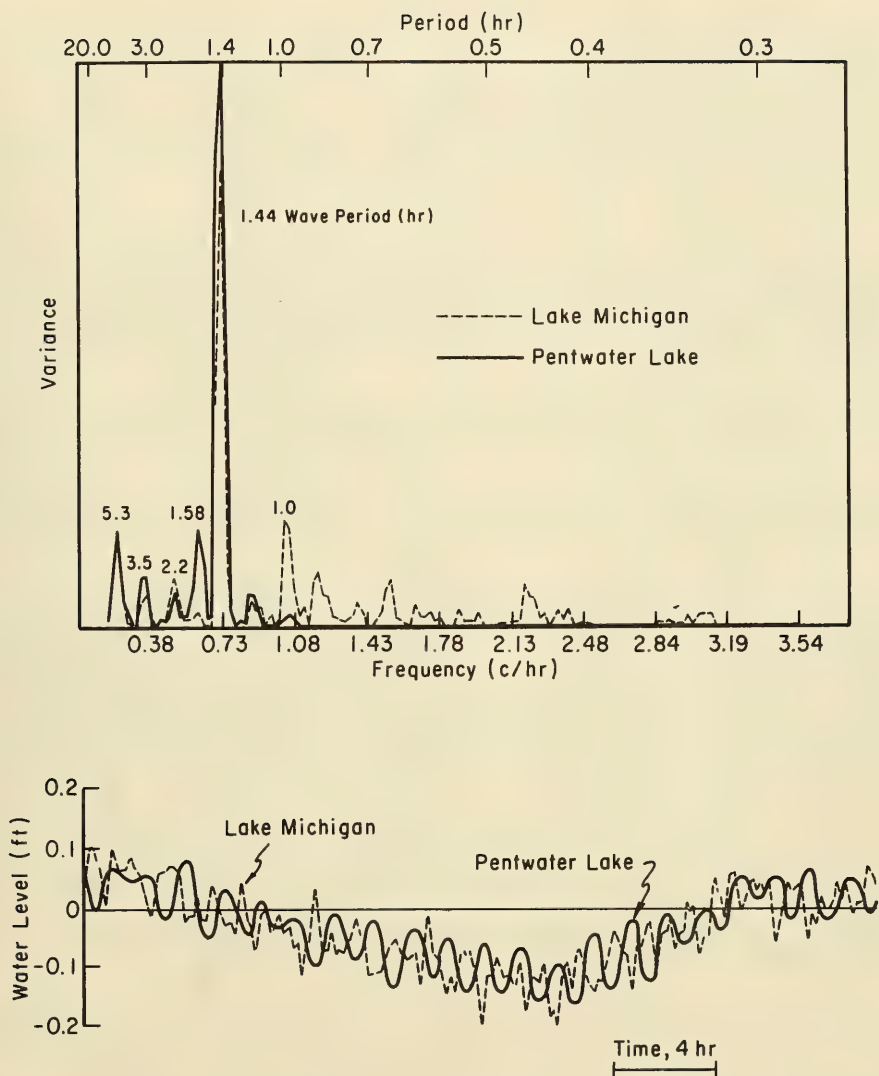


Figure 21. Sample Pentwater and Lake Michigan water levels and spectra.

with an amplification factor of 1.0, as predicted by the numerical model, because the wave period is much longer than the Pentwater Helmholtz period of 1.8 hours. However, the 0.65-hour wave, which reaches heights of 15 centimeters (0.5 foot), has a negligible effect on the harbor because it is much shorter than the Helmholtz period of 1.8 hours. Waves of 1.44 and 1.25 hours are slightly amplified by the harbor (shown by the spectral analysis in Fig. 19), but these waves are difficult to distinguish in the record because of the mixing of individual wave components.

The storm event in Figure 20 shows that a different set of modes of oscillation of Lake Michigan is present. The 1.44-hour wave is the highest, is amplified the most, and probably generates the highest percentage of significant reversing inlet current velocities. A 1.8-hour period wave is also present and is amplified. Waves shorter than 1 hour are damped by the harbor.

An unusual water level fluctuation at Pentwater where only the 1.44-hour wave is dominant in Lake Michigan, is shown in Figure 21.

As predicted previously, the 1.8- and 1.44-period waves which are the sixth and eighth longitudinal modes of oscillation of Lake Michigan, cause the highest current velocities.

Figure 22 shows the wide variation of water level fluctuations occurring in three different harbors along the eastern shore of Lake Michigan at the same time (Pentwater and Ludington are only 2.3 kilometers (11 miles) apart). The reasons for the differences are that the forcing waves outside each location are different as a result of the node-anti-node pattern of seiching in Lake Michigan (see Sec. II) and because each harbor responds differently to the forcing that is present (see Sec. IV); e.g., the 1.44- and 1.28-hour waves in Pentwater and Ludington are not noticeable in Muskegon harbor which has a Helmholtz period of 5 hours.

The forcing of harbors on the other Great Lakes will be completely different because the system of seiching varies from lake to lake; e.g., on Lake Superior, wave periods of 0.59, 0.68, 0.95, 1.14, and 1.7 hours occur in Little Lake Harbor (Fig. 23). Shorter period waves may also occur in Lake Superior, but are not observed in the harbor because the harbor dampens waves shorter than approximately 0.4 hour.

The 1.7-, 1.14-, and 0.95-hour waves on Lake Superior (the 7th, 10th, and 11th longitudinal modes of oscillation) were observed to cause high reversing currents and associated navigation problems at Duluth-Superior; e.g., on 10 June 1973, a 1.7-hour wave with a height of approximately 30 centimeters (1 foot) in the harbor, in conjunction with small 0.95-hour period waves, affected Duluth-Superior. Velocities as high as 200 centimeters per second (6.5 feet per second) were generated in Duluth inlet and 140 centimeters per second (4.5 feet per second) in Superior (Fig. 24). High velocities are generated in these inlets because of the large forcing waves in Lake Superior at this location which have periods

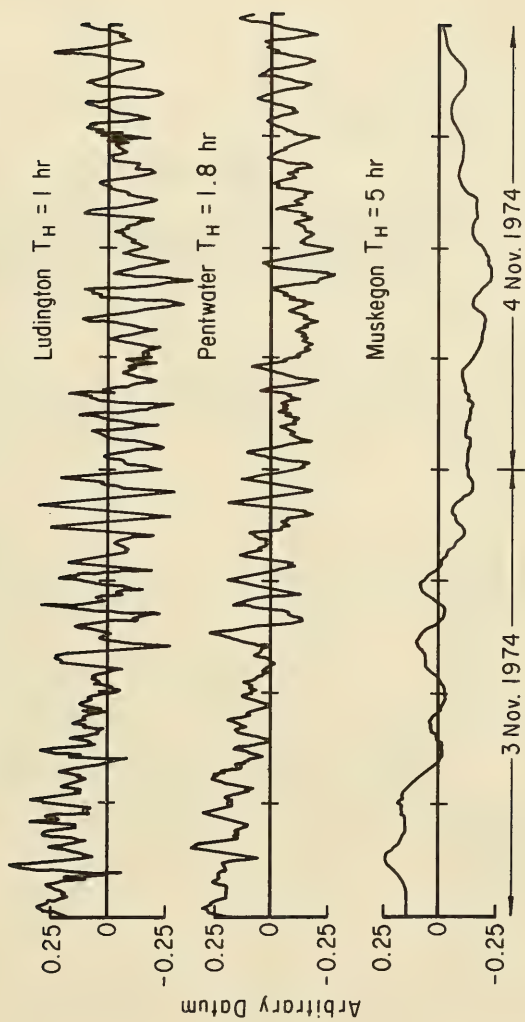


Figure 22. Sample water level fluctuations.

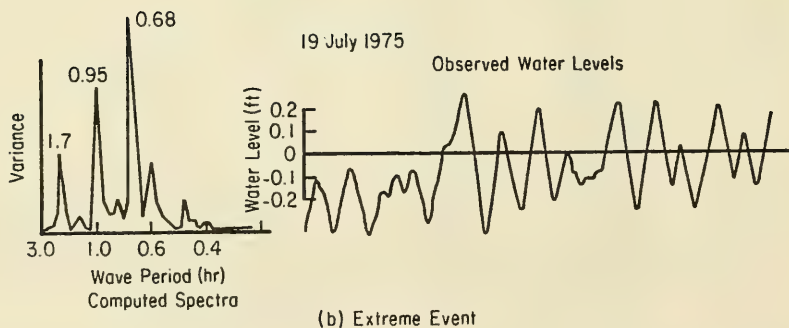
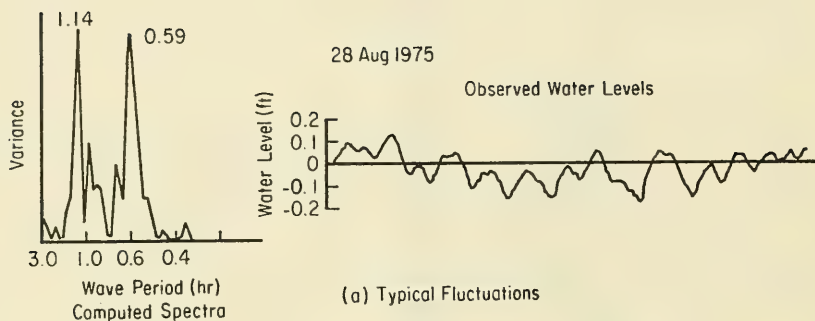


Figure 23. Water level fluctuations in Little Lake Harbor.

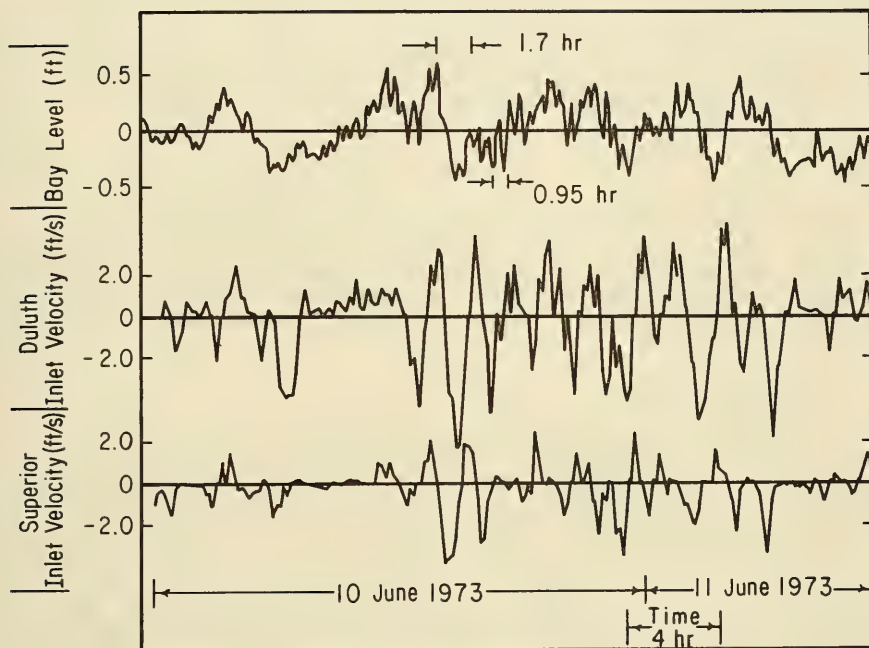


Figure 24. Sample bay levels and inlet water velocities at Duluth-Superior (inlet velocities are approximate; gages had not recently been calibrated).

near the harbor Helmholtz period. Forcing waves are large because the harbor is located on the converging end of the lake, which will always be an antinode of longitudinal oscillations.

Maximum water velocities observed in other inlets are much lower than in Duluth-Superior; e.g., at Pentwater, all measurements and predictions show that velocities are less than 60 centimeters per second (2 feet per second) for 99.5 percent of the time (Fig. 25). Predicted inlet velocities for other locations show that Portage, Ludington, and Pentwater have similar velocity distributions; Presque Isle, Muskegon, and Little Lake have still lower velocities (Fig. 26).

V. INLET DESIGN

Great Lakes inlet design problems generally fall into one of two classes: (a) a pond or lake to be connected to one of the Great Lakes by a new channel, and (b) an existing inlet channel to be modified. The concepts and techniques developed in this study can be used to aid the design of an inlet in either class. An example application for each class is given below.

1. New Inlet Channel.

The procedures for analysis of a new channel that is to connect a lake to one of the Great Lakes are: (a) determine the approximate inlet dimensions (length, width, and depth) based on physical limitations such as the desired navigable depth and width and the distance between the lake and Great Lakes; (b) estimate a Manning's n for the proposed channel (see Sec. IV, 2, for typical values of n); (c) use the numerical model to obtain monochromatic response characteristics of the harbor for the range of expected lake seiche periods and a typical amplitude; (d) compare the results to those of other nearby harbors; and (e) apply the numerical model to predict inlet velocities, discharge, and bay levels for the period of record (if Great Lakes water level fluctuation records are available in the vicinity of proposed site).

For example, suppose an inlet is to be designed to connect Crystal Lake to Lake Michigan (Fig. 27). Crystal Lake, located on the eastern shore of Lake Michigan 35 kilometers (22 miles) north of Portage Lake, has a bay surface area of 4.12×10^7 square meters (4.44×10^8 square feet). The inlet at this site would be approximately 1,200 meters (4,000 feet) long. Assume that the inlet would be 61 meters (200 feet) wide and 5.5 meters (18 feet) deep. Since the inlet is similar to Pentwater (see Table 5), the Manning's n for this channel is estimated to be 0.036.

The numerical model was run for Crystal Lake using Lake Michigan seiche periods of 9.3, 5.3, 3.5, 2.2, 1.85, and 1.4 hours with an amplitude of 3 centimeters. The predicted response characteristics of this inlet-bay system are shown in Figure 28. The model predicts that

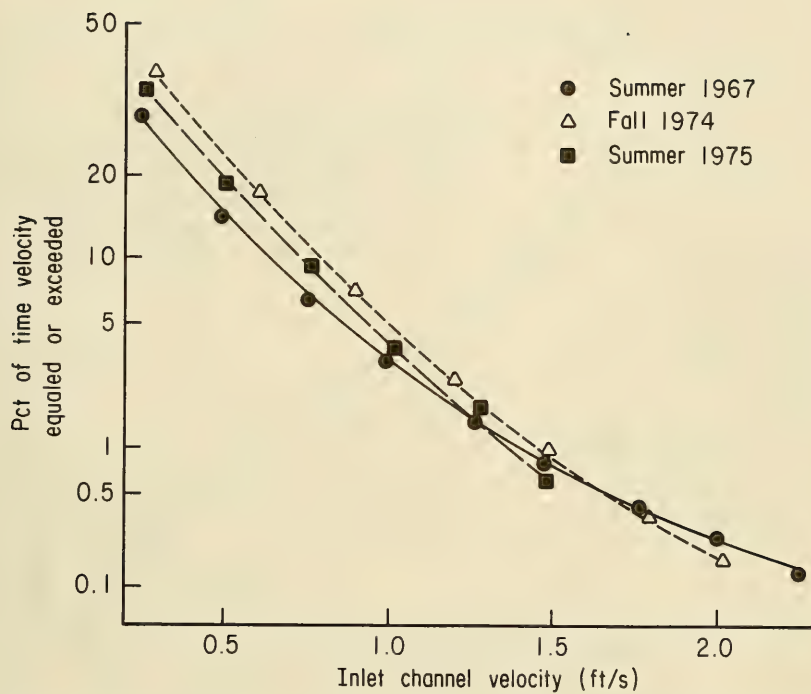


Figure 25. Pentwater inlet cumulative frequency velocity distributions (1967, 1974, and 1975).

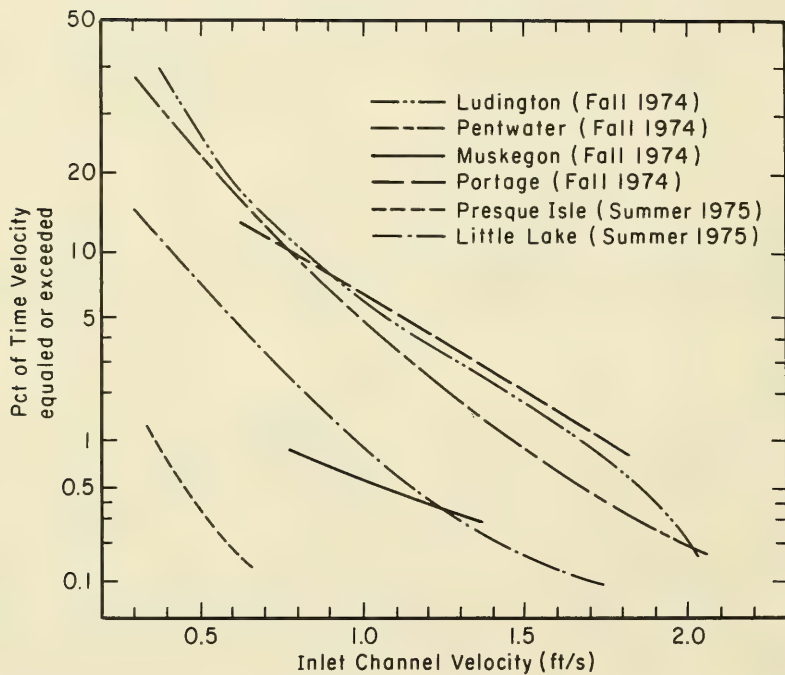


Figure 26. Inlet velocity cumulative frequency distributions (based on equation 6 and bay water level records).

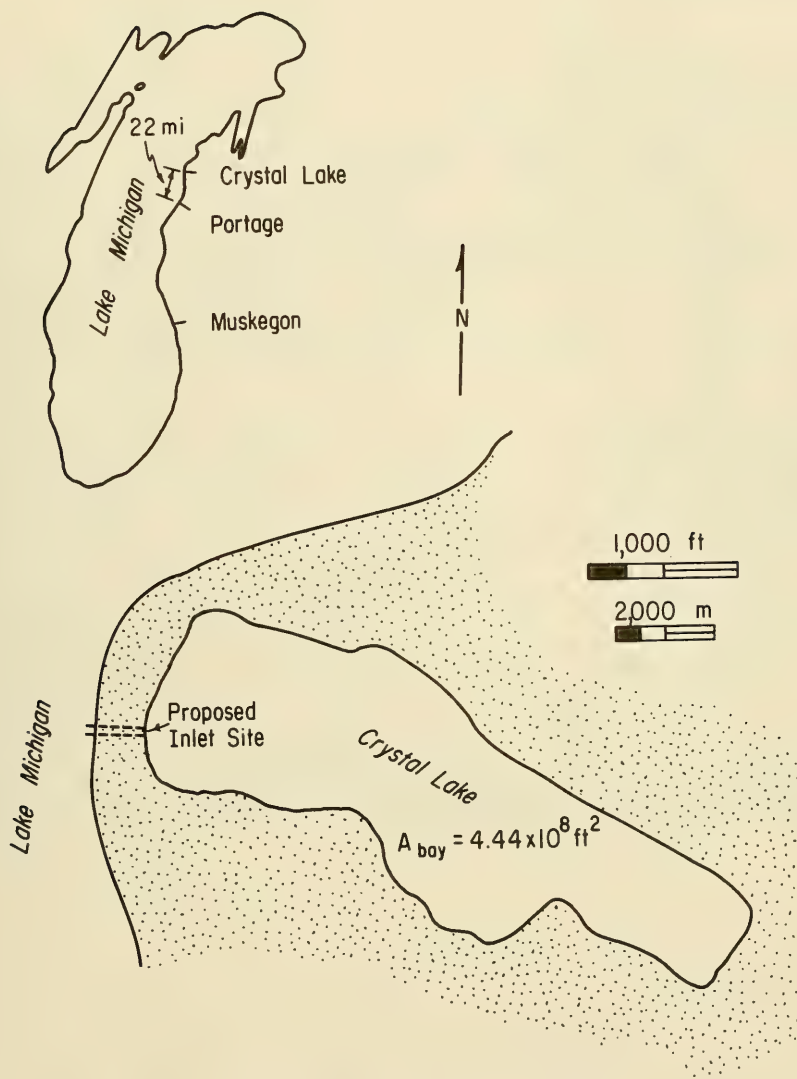


Figure 27. Crystal Lake, Michigan.

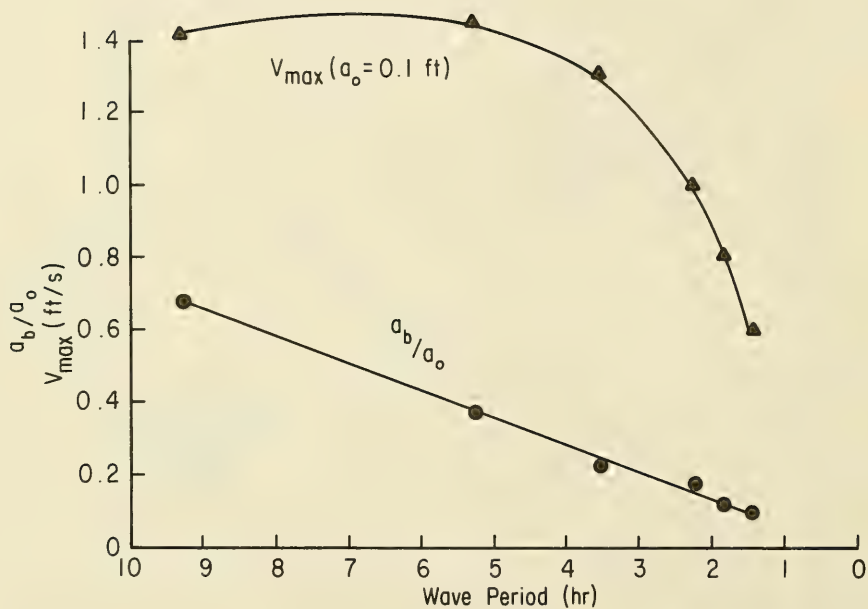


Figure 28. Predicted response characteristics of an inlet for Crystal Lake ($L = 4,000$ feet, $A = 3,600$ square feet, $D = 18$ feet, $B = 200$ feet).

the wave amplitude will be smaller in the bay than in Lake Michigan, primarily because the bay surface area is much larger than the inlet design cross-sectional area (a ratio of approximately 10^5).

The model also predicts that monochromatic seiches with an amplitude of 3 centimeters will generate maximum velocities of 43 centimeters per second (1.4 feet per second) for wave periods of 4 to 9 hours (Fig. 28). Since maximum velocities decrease for waves shorter than 4 hours, a wave with a 1-hour period produces insignificant inlet velocities.

The first three modes of oscillation of Lake Michigan (9.3-, 5.3-, and 3.5-hour waves) will generate the highest velocities for the Crystal Lake inlet design. Portage is located near Crystal Lake; therefore, forcing amplitudes of the first three modes of oscillation of Lake Michigan will be similar at both locations (Fig. 3). The predicted velocities for a given wave are different at Portage and Crystal Lake (Fig. 29) due to differences in inlet and bay geometry. However, Portage water level data can be used to estimate Crystal Lake inlet velocities.

To predict inlet velocities at Crystal Lake using Portage data, the measured Portage bay level fluctuations must first be adjusted to estimate the nearby Lake Michigan wave amplitudes. These amplitudes are then used to predict velocities at Crystal Lake; e.g., a measured Portage bay level fluctuation has a period of 3.5 hours and amplitude of 0.15 foot (4.6 centimeters). This wave was amplified by a factor of 1.3 by Portage harbor (Fig. 16), so the Lake Michigan wave amplitude was $0.15/1.3 = 0.12$ foot. A 0.1-foot wave amplitude in Lake Michigan produces a maximum velocity of 1.3 feet per second at Crystal Lake inlet (Fig. 28); therefore, the 0.12-foot wave produces $1.3 (0.12) = 1.6$ feet per second maximum velocity. This procedure could be followed for other seiche modes to estimate the maximum velocities expected at Crystal Lake.

If a complete analysis of inlet velocities is required, water levels should be measured in Lake Michigan adjacent to Crystal Lake for at least several months. These levels can be used as the forcing function in the numerical model to produce a predicted time history of inlet velocities, discharge, and bay levels for the period of record.

2. Inlet Channel Modification.

Procedures for investigating the effect of a modification to an inlet are: (a) Determine the geometry of the present system and obtain prototype hydraulic data (i.e., concurrent bay levels, Great Lakes levels, and inlet velocities); (b) calibrate the numerical model; (c) obtain monochromatic response characteristics of the inlet-bay system, (d) modify the model geometry to reflect the proposed inlet change and predict the response characteristics of the new condition; and (e) use the water level records in the Great Lakes to force the model to produce a time history of inlet velocities, discharge, and bay levels for the proposed design.

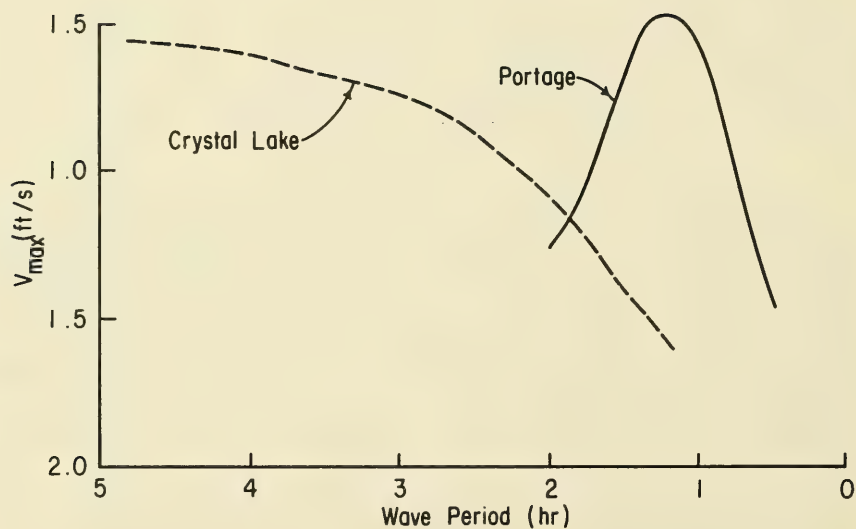


Figure 29. Predicted inlet velocities at Michigan inlets ($a_0 = 0.1$ foot).

For example, assume that a prediction of inlet velocities and the amplitude of bay level fluctuations at Pentwater is desired if (a) the inlet was deepened to 7.3 meters (24 feet), and (b) the inlet was allowed to shoal to a depth of 1.8 meters (6 feet).

The monochromatic response of the inlet-bay system for these two inlet modifications is predicted by changing the inlet geometry in the calibrated model of Pentwater. Each model was run with sinusoidal wave periods between 0.5 and 5 hours and an amplitude of 3 centimeters to predict amplification of the wave in the harbor and maximum velocity in the inlet. Figures 30 and 31 show the amplification and maximum velocities, respectively, for the 1967 inlet geometry, and for inlet depths of 1.8 and 7.3 meters. The results are summarized in Table 8.

Table 8. Summary of Pentwater hydraulic characteristics for selected inlet depths.

	D=6 ft	1967 (11 ft<D<20 ft)	D=24 ft
$(a_p/a_o)_{max}$	1.06	1.7	2.9
T_H (hr)	5.0	1.8	1.0
V_{max} (ft/s) ¹	1.0	2.0	2.7
T_{vmax} (hr) ¹	1.6	1.4	0.95

¹For $a_o = 0.1$ foot.

These predictions show that deepening the Pentwater channel from 1.8 to 7.3 meters causes the peak amplification and inlet velocity to increase, and the Helmholtz period and period of maximum velocity to decrease. Comparison of the modes of oscillation of Lake Michigan (Table 3) with the predicted velocities (Fig. 31) suggests that the 0.85-, 0.97-, 1.1-, 1.25-, and 1.44-hour waves will generate the highest reversing currents at Pentwater if the inlet was deepened to 7.3 meters.

The Lake Michigan water levels recorded on 18 August 1967 (Fig. 6) were used to force the lumped parameter model for the selected depths (Fig. 32). The model predicts that for these Lake Michigan level fluctuations, the maximum velocity for an inlet 1.8 meters deep would be 46 centimeters per second (1.5 feet per second); a 7.3-meter-deep inlet would have a velocity of 107 centimeters per second (3.5 feet per second).

VI. SUMMARY AND CONCLUSIONS

1. Meteorologically generated seiches cause most of the significant reversing currents at Great Lakes inlets. Seiche periods and node-anti-node patterns can be predicted numerically. However, water levels must

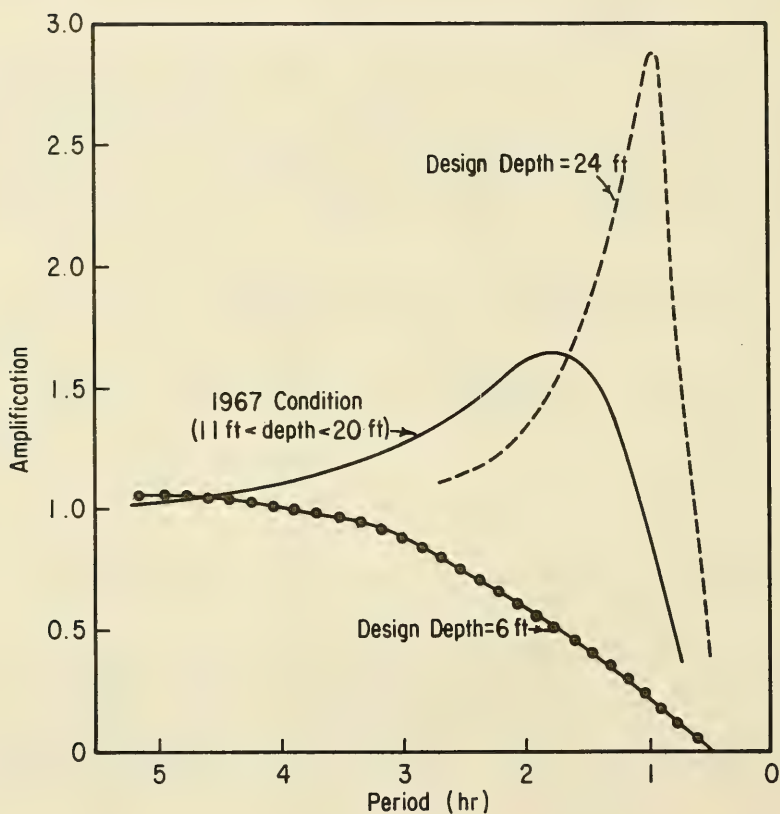


Figure 30. Response to long wave excitation at Pentwater (wave amplitude = 0.1 foot).

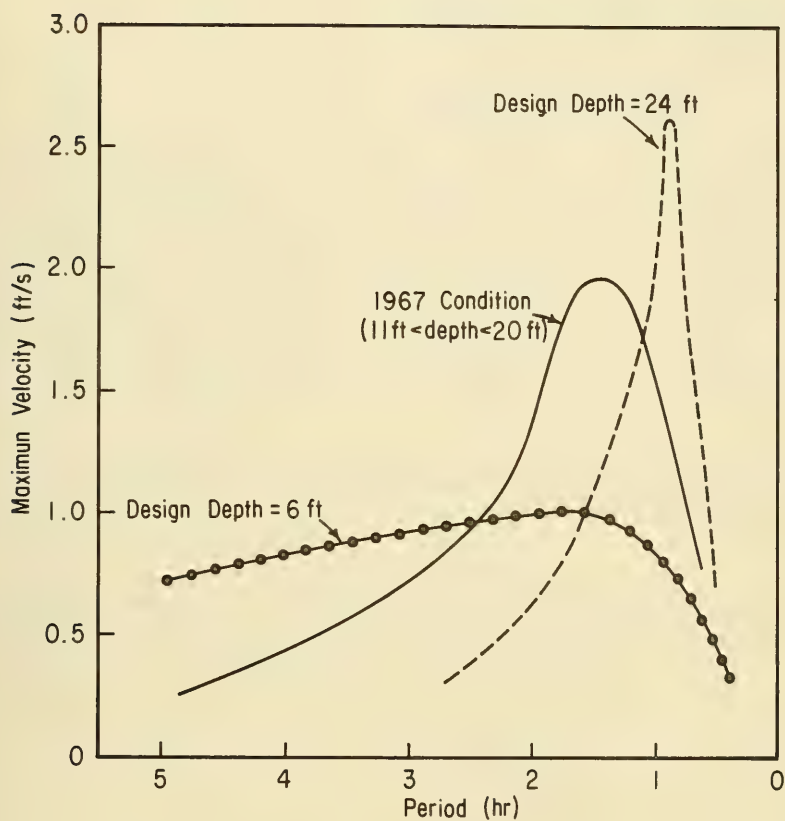


Figure 31. Response to long wave excitation at Pentwater (wave amplitude = 0.1 foot).

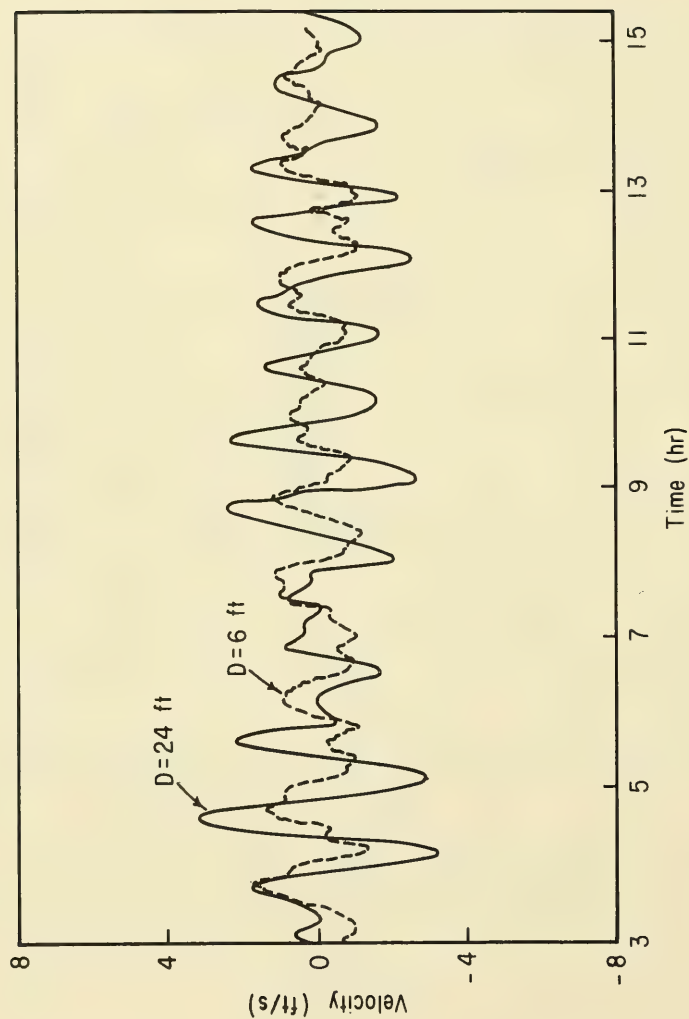


Figure 32. Predicted Pentwater inlet velocities for Lake Michigan water levels recorded on 18 August 1967 (see Fig. 6); channel depths of 6 and 24 feet.

be measured near the point of interest to obtain detailed amplitudes and periods of the active seiche modes.

2. A simple numerical model developed at the Coastal Engineering Research Center (CERC) can be used to predict inlet velocities, discharge, and bay levels for Great Lakes inlets. The model was applied to evaluate the hydraulic characteristics of several Great Lakes inlets, and examples are given using the model for typical design computations.

3. Numerical modeling of selected inlets showed that head, temporal acceleration or inertia, convective acceleration, and friction may all be important in controlling the hydraulics of Great Lakes inlets. Temporal acceleration may be especially important as it causes bay fluctuations to be amplified and out of phase with the forcing wave. As a result, a large head differential may be generated for waves with periods approximately equal to the Helmholtz period of the inlet-bay system. For a given amplitude, the highest reversing inlet currents will occur for wave periods slightly smaller than the Helmholtz period. Since even a small-amplitude seiche may generate significant reversing inlet velocities if the wave period is near the inlet-bay Helmholtz period, water levels should be carefully measured.

4. Reversing inlet currents can also be predicted by the continuity equation from high-quality bay water level records. Cumulative frequency distributions of inlet velocity developed in this manner are presented for several Great Lakes inlets.

5. Reversing velocities at most inlets are generally small. However, velocities may be high if the inlet is located where lake seiche amplitudes are relatively large and have a period approximately equal to the inlet-bay system Helmholtz period; e.g., at Duluth-Superior.

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